MATH 120–04 (Kunkle), Exam 1 Name: ______________________

100 pts, 75 minutes Sept. 9, 2014 Page 1 of 1

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers. Supporting work will be required on every problem worth more than 2 points.

You will not receive credit for using l'Hospital's Rule (a technique learned later in a calculus course) on any problem on this exam.

When asked to find $f'(x)$, you are expected to do so using the definition of derivative as a

Source limit, and not the shortcut methods learned later in a calculus course.

2.4 1 (6 pts). State the precise definition of $\lim_{x\to s} h(x) = P$

2.4 2 (8 pts). Write an ε-δ proof of $\lim_{x\to 1}(5x-3) = 2$

2.7, 2.8 3a (15 pts). Use the definition of derivative to find $f'(a)$ if $f(x) = \frac{2x}{4-x}$

2.8 3b (3 pts). Suppose that the position (in meters) of an object moving along a number line is $\frac{2x}{4-x}$ at time x (measured in seconds). Use your answer to 3a to find the object's velocity at time 6 seconds. Don't forget the units in your answer.

2.2, 2.5, 2.6, 2.7, 2.8 4 (13 pts). On the grid provided, sketch the graph of a function $f(x)$ that clearly possesses all of the following properties. Attach arrows ($\rightarrow$) to the ends of your curve to show its direction any place that it leaves the grid.

a. The domain of $f(x)$ is $(-\infty,-1)\cup(-1,6]$, that is, all $x \le 6$ except -1.

b. $\lim_{x\to -1^-} f(x) = -\infty$. c. $\lim_{x\to -1^+} f(x) = \infty$ d. $\lim_{x\to -\infty} f(x) = 2$.

e. $\lim_{x\to 0} f(x)$ does not exist. f. $\lim_{x\to 2} f(x) = -1$. g. $f(x)$ is discontinuous at $x = 2$.

h. $f(-5) = 1$. i. $f'(-5) = 0$. j. $\lim_{x\to 6^-} f'(x) = \infty$.

k. $f(-3) = 4$. l. $f'(-3) > 0$.

2.3 5 (10 pts). Evaluate the limit. One-word answers are not acceptable for full credit, but your final answer to each part should be a number, ∞, $-\infty$, or **DNE**.

$$\lim_{h\to 0}\frac{(1+h)^4-1}{h}$$

2.2, 2.3, 2.6 6 (23 pts). Evaluate the limit. One-word answers are not acceptable for full credit, but your final answer to each part should be a number, ∞, $-\infty$, or **DNE**.

a. $\displaystyle\lim_{x\to 3}\frac{\sqrt{3}-\sqrt{x}}{3-x}$ b. $\displaystyle\lim_{x\to -2}\frac{x^3-4x}{2x^2+7x+6}$

c. $\displaystyle\lim_{x\to -\infty}\frac{x^3-4x}{2x^2+7x+6}$ d. $\displaystyle\lim_{x\to \infty}\frac{x^3-4x}{2x^2+7x+6}$

2.2, 2.5 7 (22 pts). Evaluate the limit. One-word answers are not acceptable for full credit, but your final answer to each part should be a number, ∞, $-\infty$, or **DNE**.

a. $\displaystyle\lim_{x\to -1}\frac{2x+1}{(x+1)^2}$ b. $\displaystyle\lim_{x\to 2}\frac{x+1}{x-2}$

c. $\displaystyle\lim_{x\to 7}\ln|x-8|$ d. $\displaystyle\lim_{x\to 8}\ln|x-8|$

Note: "n.p." = no penalty. A small error worth less than one point.

1. For every $\varepsilon > 0$ there's a corresponding number $\delta > 0$ so that $0 < |x-8| < \delta$ implies $|h(x) - P| < \varepsilon$. $\boxed{\text{Extra credit: Problem 1 was worth 7 pts}}$

Alternate Form of this question: Replace 8 with r and P with Q.

2. Suppose $\varepsilon > 0$. Choose $\delta = \varepsilon/5$. Then $0 < |x-1| < \delta$ implies that $|5x-3-2| = |5x-5| = |5||x-1| = 5|x-1| < 5\delta = 5 \cdot \varepsilon/5 = \varepsilon$, as desired.

Alt: Let $\varepsilon > 0$. Choose $\delta = \varepsilon/7$. Then $0 < |x-1| < \delta$ implies that $|7x-3-4| = |7x-7| = |7||x-1| = 7|x-1| < 7\delta = 7\frac{\varepsilon}{7} = \varepsilon$, as desired.

3a. $f'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim\limits_{h \to 0} \frac{\frac{2(a+h)}{4-(a+h)} - \frac{2a}{4-a}}{h} \cdot \frac{(4-a-h)(4-a)}{(4-a-h)(4-a)}$

$= \lim\limits_{h \to 0} \frac{2(a+h)(4-a) - 2a(4-a-h)}{h(4-a-h)(4-a)} = \lim\limits_{h \to 0} \frac{2(4a+4h-a^2-ah) - "}{"}$

$= \lim\limits_{h \to 0} \frac{8a+8h-2a^2-2ah-8a+2a^2+2ah}{h(4-a-h)(4-a)} = \lim\limits_{h \to 0} \frac{8h}{h(4-a-h)(4-a)}$

$= \lim\limits_{h \to 0} \frac{8}{(4-a-h)(4-a)} = \frac{8}{(4-a)(4-a)} = \frac{8}{(4-a)^2}$.

Alt: Replace 4 by 6 $\Rightarrow$ Replace 8 by 12. ans: $f'(a) = \frac{12}{(6-a)^2}$

3b. Velocity $= f'(6) = \frac{8}{(-2)^2} = 2$ m/sec. Alt: $f'(8) = \frac{12}{(-2)^2} = 3$ m/sec

4 (13 pts). On the grid provided, sketch the graph of a function $f(x)$ that clearly possesses all of the following properties. Attach arrows ($\to$) to the ends of your curve to show its direction any place that it leaves the grid.

a. The domain of $f(x)$ is $(-\infty, -1) \cup (-1, 6]$, that is, all $x \leq 6$ except -1.

b. $\lim_{x \to -1^-} f(x) = \infty$.

c. $\lim_{x \to -1^+} f(x) = -\infty$

d. $\lim_{x \to -\infty} f(x) = -2$.

e. $\lim_{x \to 0} f(x)$ does not exist.

f. $\lim_{x \to 2} f(x) = 1$.

g. $f(x)$ is discontinuous at $x = 2$.

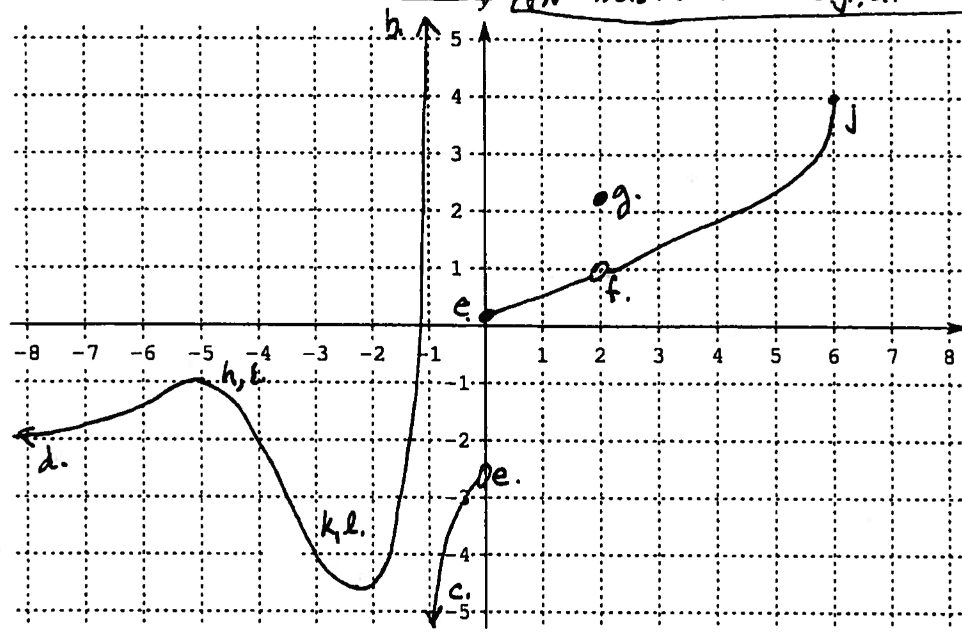
h. $f(-5) = -1$.

i. $f'(-5) = 0$.

j. $\lim_{x \to 6^-} f'(x) = \infty$.

k. $f(-3) = -4$.

l. $f'(-3) < 0$. These parts were thrown out. n.p. for mistakes on i, j, l.



Alt :

a. The domain of $f(x)$ is $(-\infty, -1) \cup (-1, 6]$, that is, all $x \leq 6$ except -1.

b. $\lim_{x \to -1^-} f(x) = -\infty$.

c. $\lim_{x \to -1^+} f(x) = \infty$

d. $\lim_{x \to -\infty} f(x) = 2$.

e. $\lim_{x \to 0} f(x)$ does not exist.

f. $\lim_{x \to 2} f(x) = -1$.

g. $f(x)$ is discontinuous at $x = 2$.

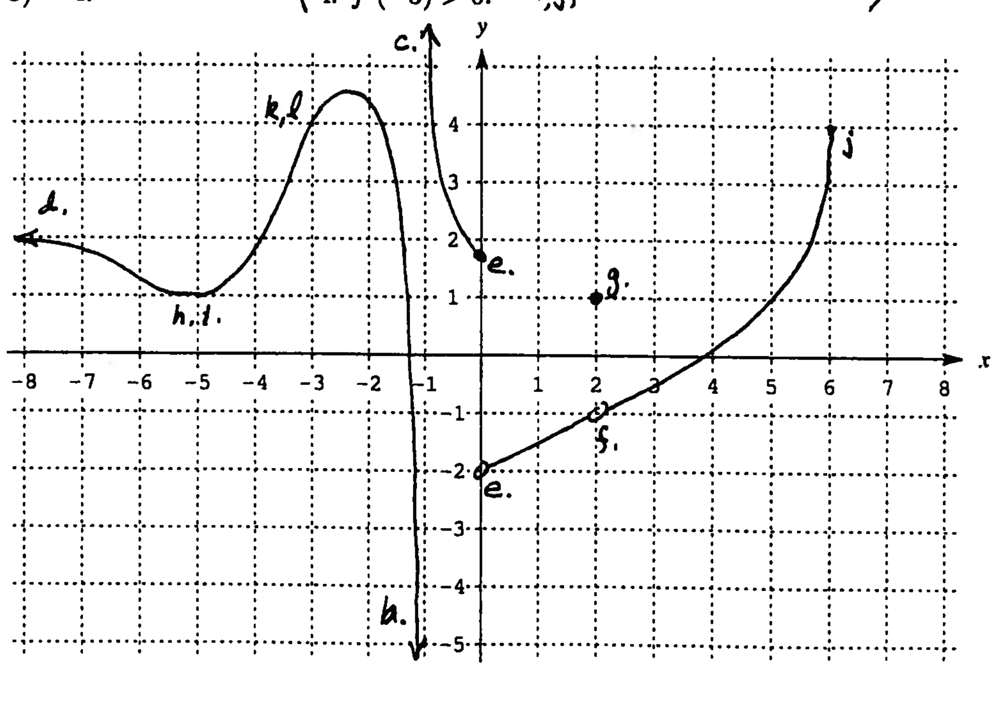
h. $f(-5) = 1$.

i. $f'(-5) = 0$.

j. $\lim_{x \to 6^-} f'(x) = \infty$.

k. $f(-3) = 4$.

l. $f'(-3) > 0$. i, j, l thrown out



5. Use Pascal's triangle:

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \end{array}$$

$$\lim_{h\to 0} \frac{(1+h)^4 - 1}{h} = \lim_{h\to 0} \frac{\cancel{1} + 4h + 6h^2 + 4h^3 + h^4 - \cancel{1}}{h} = \lim_{h\to 0} \frac{\cancel{h}(4+6h+4h^2+h^3)}{\cancel{h}\cdot 1} = 4.$$

6. a. $\lim_{x\to 3} \frac{(\sqrt{3}-\sqrt{x})}{(3-x)} \cdot \frac{(\sqrt{3}+\sqrt{x})}{(\sqrt{3}+\sqrt{x})} = \lim_{x\to 3} \frac{\cancel{(3-x)}\cdot 1}{\cancel{(3-x)}(\sqrt{3}+\sqrt{x})} = \frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}}.$

$\boxed{7\text{PTS}}$

Alt: Replace 3 throughout by 5.

b. $\lim_{x\to -2} \frac{x^3-4x}{2x^2+7x+6} = \lim_{x\to -2} \frac{x(x-2)(x+2)}{(x+2)(2x+3)} = \lim_{x\to -2} \frac{x(x-2)}{(2x+3)} = \frac{(-2)(-4)}{-1} = -8.$

c. Solution 1: $\text{limit} = \lim_{x\to -\infty} \frac{\text{lead term}}{\text{lead term}} = \lim_{x\to -\infty} \frac{x^3}{2x^2} = \lim_{x\to -\infty} \frac{x}{2} = -\infty.$

Solution 2: $\lim_{x\to -\infty} \frac{x^3(1-\frac{4}{x^2})}{x^2(2+\frac{7}{x}+\frac{6}{x^2})} = \lim_{x\to -\infty} \frac{\overset{\nearrow -\infty}{x}\,\overset{\nearrow 1}{(1-\frac{4}{x^2})}}{\underset{\searrow 2}{(2+\frac{7}{x}+\frac{6}{x^2})}} = -\infty.$

5. d. As in c, $\text{limit} = \lim_{x\to +\infty} \frac{x}{2} = \infty.$

Alt form: c. d..

$\boxed{\text{c. \& d. were worth 9 pts together}}$

7. a. as $x\to -1$, $\frac{2x+1}{(x+1)^2} \to$ "$\frac{-1}{0}$" $\Rightarrow$ Blow Up. Only possible limits are $\pm\infty$.

$\boxed{7\text{PTS}}$

Check signs: $\frac{2x+1}{(x+1)^2} = \frac{-}{+}\} -$. Conclude limit $= -\infty$.

b. as $x\to 2$, $\frac{x+1}{x-2} \to$ "$\frac{3}{0}$" $\Rightarrow$ Blow up. Sign? $\frac{x+1}{x-2} = \frac{+}{\pm}$, depending on $x\to 2^{\pm}$

Conclusion: $\lim_{x\to 2^+}$ and $\lim_{x\to 2^-}$ are $\pm\infty$. $\lim_{x\to 2}$ does not exist.

Alt: a. b.

c. As $x\to 7$, $\ln|x-8| \to \ln|-1| = \ln 1 = 0$ (Continuity of ln allows us to find this limit by evaluating the function.)

d. As $x\to 8$, $|x-8|\to 0^+$, so $\ln|x-8| \to -\infty$. (Recall graph of $\ln x$:)

Alt: c. d.

$\boxed{\text{c. d. were worth 4 pts each.}}$