Read each problem carefully. No calculators, notes, books, or any outside materials. Unless otherwise indicated, **supporting work will be required on every problem**; one-word answers, or answers which simply restate the question, will receive no credit. You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$. If you wish, you can leave unfinished arithmetic in your final answers. You will not receive credit for using l'Hospital's Rule on any problem on this exam.

1 (18 pts). Two sides of a triangle have lengths 4cm and 3cm. The angle between them is increasing at the rate of 2 radians/sec. How fast is the length of the third side increasing when the angle between the two sides of fixed length is $5\pi/6$?

2a (8 pts). Find the linearization of $e^{2x} + e^{-5x}$ at $a = 0$.

2b (4 pts). Use your answer to approximate $e^{0.2} + e^{-0.5}$.

3a (2 pts). Find the domain of $h(x) = x \ln x + 3x$

3b (10 pts). Find the critical points of $h(x) = x \ln x + 3x$

4. Let $g(x) = 4x^3 + 3x^2 - 6x + 12$.
   a (10 pts). On what interval(s) is $g(x)$ increasing?
   
   b (2 pts). At what $x$-value(s) does $g$ have a local minimum?
   
   c (2 pts). At what $x$-value(s) does $g$ have a local maximum?
   
   d (8 pts). On what interval(s) is the graph of $g(x)$ concave up?
   
   e (2 pts). At what $x$-value(s) does $g$ have an inflection point?

5 (8 pts). State Rolle's Theorem:
   
   If _____________________________________________________________
   
   then ____________________________________________________________
   ________________________________

6 (8 pts). Explain why, for any real number $N$, the equation $x^3 + 12x + N = 0$ cannot have two real solutions.

7 (18 pts). Find the absolute maximum and absolute minimum of $r(x) = x\sqrt{10-x^2}$ on the interval $[-\sqrt{10}, 1]$. 
1. See figure. Given \( \frac{d\theta}{dt} = 2 \). Want \( \frac{dc}{dt} \)

Laws of Cosines:

\[ c^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos \theta \]

Differentiate wrt time:

\[ 2c \frac{dc}{dt} = 24 \sin \theta \frac{d\theta}{dt} \]

At moment in question, \( \theta = \frac{5\pi}{6} \). \( \sin \frac{5\pi}{6} = \frac{1}{2} \); \( \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} \);

\[ c = \sqrt{9 + 16 - 24 \cdot \left( -\frac{\sqrt{3}}{2} \right)} = \sqrt{25 + 12\sqrt{3}} \]

Plug in and solve for \( \frac{dc}{dt} \):

\[ \frac{dc}{dt} = \frac{24 \left( \frac{1}{2} \right) (2)}{2 \sqrt{25 + 12\sqrt{3}}} \]

more done

Alternate form; \( \frac{d\theta}{dt} = 3 \): \( \frac{dc}{dt} = \frac{24 \left( \frac{1}{2} \right) \cdot 3}{2 \sqrt{25 + 12\sqrt{3}}} \)

2a. \( f(x) = \frac{x^2}{x} + \frac{5}{x} \). \( f'(x) = 2e^{2x} - 5e^{5x} \)

\( f(1) = 2 \); \( f'(1) = 2 - 5 = -3 \). \( L(x) = f(1) + f'(1)(x - 1) = 2 - 3(x - 1) \)

2b. \( e^{2x} + e^{5x} \Rightarrow f(0,1) = L(0,1) = 2 - 3(0,1) \) done

Alternate

2a. \( f(0) = 2 \); \( f'(0) = 2 - 0 = -4 \). \( L(x) = 2 - 4x \)

2b. \( f(0,1) = L(0,1) = 2 - 4(0,1) = 1,0 \)

3a. Domain \( x \cdot \ln x + 3x \) is \( (0, \infty) \)

3b. Critical points:

\[ h'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} + 3 = \ln x + 4 \]

(\( h'(x) \) is defined for all \( x \) in domain \( h \), so only critical points satisfy \( dh'(x) = 0 \))

\[ h'(x) = 0 \Rightarrow \ln x + 4 = 0 \Rightarrow e^{\ln x} = e^{-4} \Rightarrow x = e^{-4} \]

Alternate form:

\[ h' = 1 \cdot \ln x + x \cdot \frac{1}{x} + 2 = \ln x + 3 \]

Critical points: \( 0 = \ln x + 3 \Rightarrow x = e^{-3} \)
4. \( g(x) = 4x^3 + 3x^2 - 6x + 12 \)

\[ g'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x-1)(x+1) \]

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>( \frac{1}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g' )</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

\( g \) increases on \((-\infty, -\frac{1}{2})\) and \((\frac{1}{2}, \infty)\)\

Act. Term: \( g'(x) = 12x^2 - 6x - 6 = 6(2x+1)(x-1) \)

<table>
<thead>
<tr>
<th>x</th>
<th>-( \frac{1}{2} )</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g' )</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

\( g \) increases on \((-\infty, -\frac{1}{2})\) and \((1, \infty)\)

4b. \( g \) has a local min @ \(-1\) (alt \(-\frac{1}{2}\))

4c. \( g \) has a local max @ \( \frac{1}{2} \) (alt 1)

4d. \( g'' = 24x + 6 = 24(x + \frac{1}{4}) \)

<table>
<thead>
<tr>
<th>x</th>
<th>(-\frac{1}{4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g'' )</td>
<td>+</td>
</tr>
</tbody>
</table>

\( g \) concave up on \((-\frac{1}{4}, \infty)\)

\( g'' = 24x - 6 = 24(x - \frac{1}{4}) \), \( g \) concave up on \((\frac{1}{4}, \infty)\).

4e. \( g \) has an inflection point @ \( x = \frac{1}{4} \) (alt \( \frac{1}{4} \))

5. **Rolle's Theorem**: If \( f \) is continuous on \([a,b]\) and differentiable on \((a,b)\), and if \( f(a) = f(b) = 0 \), then \( f'(c) = 0 \) for some \( c \) in \((a,b)\).

Alternate form:

**Mean Value Theorem**: If \( f \) is continuous on \([a,b]\) and differentiable on \((a,b)\), then 
\[ f(b) - f(a) = f'(c)(b-a) \] for some \( c \) in \((a,b)\).
6. Here are two solutions:

1. \( f(x) = x^3 + 12x + N \). \( f'(x) = 3x^2 + 12 \). \( f'(x) \) never = 0.

   Since \( f \) is continuous and differentiable everywhere, Rolle's Theorem says that between any two zeros of \( f(x) \) there must be a zero of \( f'(x) \). Since \( f(x) \) has no zeros, \( f \) cannot have 2 zeros.

2. \( f'(x) = 3x^2 + 12 \) is always > 0, so \( f \) is always increasing. Since an increasing function can't cross the x-axis more than once, \( f(x) \) can't have 2 zeros.

7. \( r(x) = x(10 - x^2)^{1/2} \). \( r'(x) = (10 - x^2)^{1/2} + x \cdot \frac{1}{2} (10 - x^2)^{-1/2}(-2x) \)

   \( r'(x) \) is undefined only when \( 10 - x^2 = 0 \), or \( x = \pm \sqrt{10} \).

   \( r'(x) = 0 \) when \((10 - x^2)^{1/2} - x^2 (10 - x^2)^{-1/2} = 0 \). Factor:

   \( (10 - x^2)^{1/2} (10 - x^2 - x^2) = 0 \). First factor never = 0. 2nd = 0 at \( x^2 = 5 \), or \( x = \pm \sqrt{5} \). Only \( x = -\sqrt{5} \) is in \((-\sqrt{10}, 1)\).

   Check \( r \) at endpoints and critical points:

   \( r(-\sqrt{10}) = 0 \)
   \( r(-\sqrt{5}) = -\sqrt{5} (10 - 5)^{1/2} = -5 \)
   \( r(1) = 1 (10 - 1)^{1/2} = \sqrt{9} = 3 \)

   abs. max is 3
   abs. min is -5.