More problems for section 3.5 of *Essentials of Precalculus with Calculus Previews* by Zill and Dewar, 5e.

To find the asymptotes for the graph of a rational function, first check for factors that are common to the numerator and denominator. Cancel these if you find any, and remove the new function's value at the zeros of the common factors you canceled. Unless the new function has a VA at these x-values, this removal creates holes in the graph.
After canceling any common factors from the top and bottom, use these rules. The graph of the the rational function $\dfrac{p(x)}{q(x)}$ has a vertical asymptote at those x-values causing $q(x) = 0$. Furthermore,

$$\frac{p(x)}{q(x)} \text{ has a } \left\{\begin{matrix}\text{horizontal}\\ \text{slant}\end{matrix}\right\} \text{ asymptote if the degree of } p \left\{\begin{matrix}\leq \text{degree of } q\\ = 1 + \text{degree of } q\end{matrix}\right\}.$$

Note that it is impossible for a rational function to have both a slant and a horizontal asymptote. Find HA and SA as in class.

1. Find all asymptotes for the graphs of the following rational functions. Find the coordinates of any holes, and any points where the function's graph intersects its HA or SA.

a. $r(x) = \dfrac{(2x+1)(x+1)}{(x-1)(x+2)}$ b. $r(x) = \dfrac{3x^2+8x+4}{x^2-3x-4}$ c. $r(x) = \dfrac{x^2+3x-4}{3x^2+7x+2}$

d. $r(x) = \dfrac{2x+1}{3x+1}$ e. $r(x) = \dfrac{2x+1}{3-x}$ f. $r(x) = \dfrac{2x+3}{(3x+2)(2x+1)}$

g. $r(x) = \dfrac{8x-1}{x}$ h. $r(x) = \dfrac{x+2}{9-x^2}$ i. $r(x) = \dfrac{x+1}{2x^2+5x+2}$

j. $r(x) = \dfrac{x-4}{x^2-16}$ k. $r(x) = \dfrac{x+4}{x^2+16}$ l. $r(x) = \dfrac{x+4}{x^2+2x-3}$

m. $r(x) = \dfrac{4-x}{3x+1}$ n. $r(x) = \dfrac{3-x}{8x^2-6x+1}$ o. $r(x) = \dfrac{2x^2-1}{x+2}$

p. $r(x) = \dfrac{4x^2-6x+4}{2x-1}$ q. $r(x) = \dfrac{x^3-1}{x^2-3}$ r. $r(x) = \dfrac{x^2+4}{3x-1}$

s. $r(x) = \dfrac{2x^2+3x+1}{x-1}$ t. $r(x) = \dfrac{3x^3-2x}{3x^2-1}$ u. $r(x) = \dfrac{4x^2+2x-1}{x+3}$

v. $r(x) = \dfrac{2x^2+3x+1}{x^2+3x+2}$ w. $r(x) = \dfrac{6x^2-x-1}{2x^2+7x-4}$ x. $r(x) = \dfrac{x^2-2x-8}{x^2-4}$

y. $r(x) = \dfrac{2x^2+7x+3}{x+3}$ z. $r(x) = \dfrac{x-2}{2x^2-8}$ 27. $r(x) = \dfrac{2x-1}{2x^2+9x-5}$

Answers

1a. $y = 2, x = 1, x = -2$. intersection: $(-5, 2)$ 1b. $y = 3, x = -1, x = 4$. intersection: $(-16/17, 3)$ 1c. $y = 1/3, x = -2, x = -1/3$. intersection: $(7, 1/3)$ 1d. $y = 2/3, x = -1/3$ 1e. $y = -2, x = 3$ 1f. $y = 0, x = -2/3, x = -1/2$ intersection: $(-3/2, 0)$ 1g. $y = 8, x = 0$ 1h. $y = 0, x = 3, x = -3$ intersection: $(-2, 0)$ 1i. $y = 0, x = -2, x = -1/2$ intersection: $(-1, 0)$ 1j. $y = 0, x = -4$ hole: $(4, 1/8)$. 1k. $y = 0$ intersection: $(-4, 0)$ 1l. $y = 0, x = -3, x = 1$ intersection: $(-4, 0)$ 1m. $y = -1/3, x = -1/3$ 1n. $y = 0, x = 1/2, x = 1/4$ intersection: $(3, 0)$ 1o. $y = 2x-4, x = -2$ 1p. $y = 2x-2, x = 1/2$ 1q. $y = x, x = \sqrt{3}, x = -\sqrt{3}$ ins: $(1/3, 1/3)$ 1r. $y = x/3 + 1/9, x = 1/3$ 1s. $y = 2x+5, x = 1$ 1t. $y = x, x = -1/\sqrt{3}, x = 1/\sqrt{3}$ ins: (0,0 1u. $y = 4x-10, x = -3$ 1v. $y = 2, x = -2$ hole: $(-1, -1)$ 1w. $y = 3, x = -4$ hole: $(1/2, 5/9)$ 1x. $y = 1, x = 2$ hole: $(-2, 3/2)$ 1y. $y = 2x+1$ hole: $(-3, -5)$ intersection: except for the hole, the entire curve is its SA. 1z. $y = 0, x = -2$ hole: $(2, 1/8)$ 127. $y = 0, x = -5$ hole:$(1/2, 2/11)$