More problems for section 2.6 of *Essentials of Precalculus with Calculus Previews* by Zill and Dewar, 6e.

The **intersection** of two sets consists of all points the two sets have in common. The domain of $f + - \times g$ is the intersection of the domain of f and the domain of g (even if $f + - \times g$ simplifies to a function with a larger domain). The domain of $f \div g$ is the intersection of the domain of f and the domain of g, excluding any points where $g = 0$ (even if ...).

1. Find the domain for $f + g$, $f - g$, $f \times g$, and $f \div g$.

a. $f(x) = \dfrac{1}{x-1}$, $g(x) = \dfrac{2}{x^2 - 2x + 3}$

b. $f(x) = \dfrac{1}{x^2 - 1}$, $g(x) = \dfrac{x}{x+1}$

c. $f(x) = \dfrac{1}{x^2 - 9}$, $g(x) = \sqrt{4 - x}$

d. $f(x) = \dfrac{1}{x^2 - 4}$, $g(x) = \sqrt{1 + x}$

e. $f(x) = \dfrac{1}{x^2 - 4x + 12}$, $g(x) = \sqrt{x - 8}$

f. $f(x) = \dfrac{1}{\sqrt{x-2}}$, $g(x) = \dfrac{x-3}{\sqrt{1+x}}$

g. $f(x) = \sqrt{2 - x}$, $g(x) = \sqrt{4 + x}$

h. $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{8 + 2x - x^2}$

i. $f(x) = \sqrt{-3 - x}, g(x) = \sqrt{-x^2 + 2x + 8}$

j. $f(x) = \dfrac{1}{x+10}, g(x) = \sqrt{27 - 6x - x^2}$

2. Find $\big(f \circ g\big)(x)$ and $\big(g \circ f\big)(x)$ and the domain of each.

a. $f(x) = \sqrt{x+2}$, $g(x) = \sqrt{2 - x}$

b. $f(x) = \sqrt{3 - x}$, $g(x) = \sqrt{1 - x}$

c. $f(x) = \sqrt{3 + x}$, $g(x) = \sqrt{2 + x}$

d. $f(x) = \sqrt{1 + x}$, $g(x) = x^2 - 4$

e. $f(x) = \sqrt{1 - x}$, $g(x) = 2x^2 + 3$

f. $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$, $g(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

g. $f(x) = \begin{cases} -1 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$, $g(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$

h. $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$, $g(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ |x| & \text{if } x < 0 \end{cases}$

i. $f(x) = \dfrac{1}{x+1}$, $g(x) = \dfrac{1}{x-1}$

j. $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{x+2}{x}$

k. $f(x) = \dfrac{x}{x+1}$, $g(x) = \dfrac{x}{x-1}$

l. $f(x) = x^2 + 2x$, $g(x) = \sqrt{x - 3}$

m. $f(x) = x^2 + 2x$, $g(x) = \sqrt{3 - x}$

Answers

1a. $f + - \times g : \mathbb{R} \setminus \{1, -1, 3\}. f \div g$: same. 1b. $f + - \times g : \mathbb{R} \setminus \{1, -1\}. f \div g \mathbb{R} \setminus \{1, 0, -1\}$. 1c. $f + - \times g : (-\infty, -3) \cup (-3, 3) \cup (3, 4]. f \div g : (-\infty, -3) \cup (-3, 3) \cup (3, 4)$ 1d. $\pm\times : [-1, 2) \cup (2, \infty)$, $\div : (-1, 2) \cup (2, \infty)$ 1e. $\pm\times : [8, \infty)$, $\div : (8, \infty)$ 1f. $\pm\times : (2, \infty)$, $\div : (2, 3) \cup (3, \infty)$ 1g. $\pm\times : [-4, 2]$, $\div : (-4, 2]$ 1h. $\pm\times : [-1, 4]$, $\div : [-1, 4)$ 1i. $\pm\times : (-\infty, -3] \cup [4, \infty)$, $\div : (-\infty, -3] \cup (4, \infty)$ 1j. $\pm\times : [-9, 3]$, $\div : (-9, 3)$ 2a. $f \circ g = \sqrt{2 + \sqrt{2 - x}}$, dom $= (\infty, 2]$; $g \circ f = \sqrt{2 - \sqrt{x + 2}}$, dom $= [-2, 2]$. 2b. $f \circ g = \sqrt{3 - \sqrt{1 - x}}$, dom $= [-8, 1]$; $g \circ f = \sqrt{1 - \sqrt{3 - x}}$, dom $= [2, 3]$. 2c. $f \circ g = \sqrt{3 + \sqrt{2 + x}}$, dom $= [-2, \infty)$; $g \circ f = \sqrt{2 + \sqrt{3 + x}}$, dom $= [-3, \infty)$. 2d. $f \circ g = \sqrt{x^2 - 3}$, dom $= (-\infty, \sqrt{3}] \cup [\sqrt{3}, \infty)$; $g \circ f = x - 3$, dom $= [-1, \infty)$. 2e. $f \circ g = \sqrt{-2 - 2x^2}$, dom $= \emptyset$; $g \circ f = 5 - 2x$, dom $= (-\infty, 1]$. 2f. $f \circ g = f$, dom $= \mathbb{R}$; $g \circ f = 0$, dom $= \mathbb{R}$. 2g. $f \circ g = -f$, dom $= \mathbb{R}$; $g \circ f = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$, dom $= \mathbb{R}$. 2h. $f \circ g = g$, dom $= \mathbb{R}$; $g \circ f = 1$, dom $= \mathbb{R}$. 2i. $f \circ g = \frac{x-1}{x}$, dom $= \mathbb{R} \setminus \{0, 1\}$; $g \circ f = -\frac{x+1}{x}$, dom $= \mathbb{R} \setminus \{-1, 0\}$. 2j. $f \circ g = \frac{x}{x+2}$, dom $= \mathbb{R} \setminus \{0, -2\}$; $g \circ f = 1 + 2x$, dom $= \mathbb{R} \setminus \{0\}$. 2k. $f \circ g = \frac{x}{2x-1}$, dom $= \mathbb{R} \setminus \{\frac{1}{2}, 1\}$; $g \circ f = -x$, dom $= \mathbb{R} \setminus \{-1\}$. 2l. $f \circ g = x - 3 + 2\sqrt{x - 3}$, dom $= [3, \infty)$; $g \circ f = \sqrt{x^2 + 2x - 3}$, dom $= (\infty, -3] \cup [1, \infty)$. 2m. $f \circ g = 3 - x + 2\sqrt{3 - x}$, dom $= (-\infty, 3]$; $g \circ f = \sqrt{3 - 2x - x^2}$, dom $= [-3, 1]$.