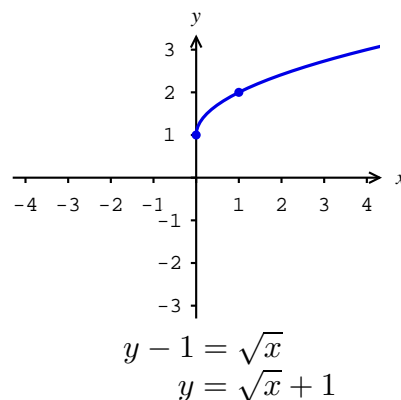
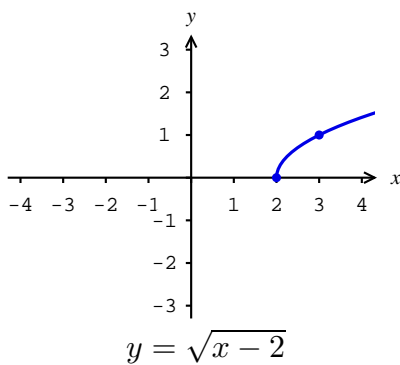
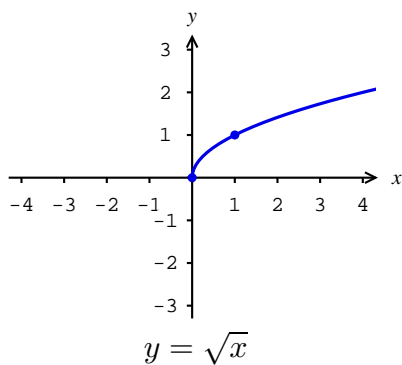
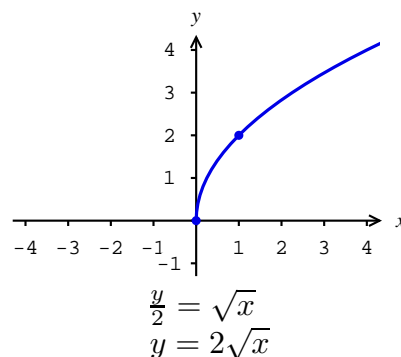
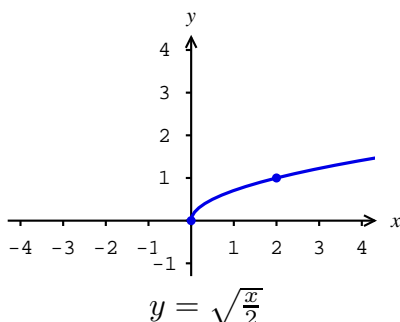
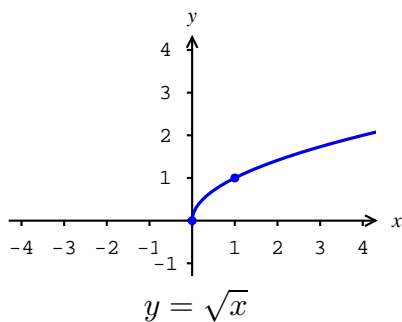
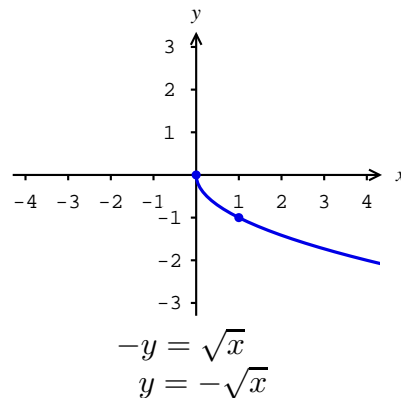
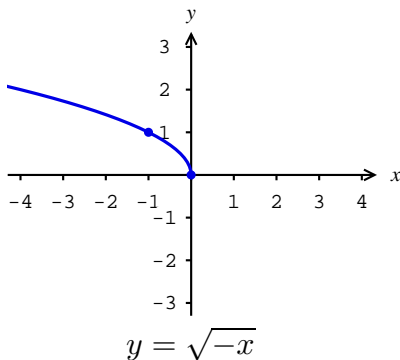
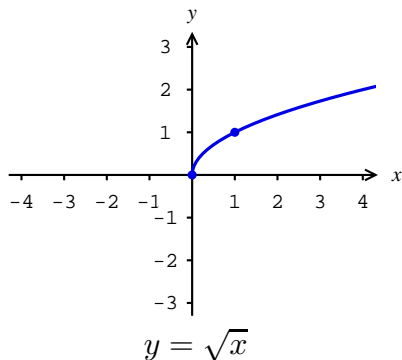


### Transformations on the graphs of equations

To understand how changes to the equation change that equation's graph, let's look at some examples using  $y = \sqrt{x}$ . Note the points  $(0, 0)$  and  $(1, 1)$  on its graph.



Since the equation  $y = \sqrt{x}$  has the form  $y = f(x)$ , the equations in the third column that result from replacing  $y$  by some other expression could also have been obtained instead by replacing the function  $f(x)$  by some other function.

These figures illustrate the following rules. Suppose  $E(x, y)$  is an equation in  $x$  and  $y$ .

Reflections:

1. The graph of  $E(-x, y)$  is obtained by reflecting the graph of  $E(x, y)$  across  $x = 0$ .
2. The graph of  $E(x, -y)$  is obtained by reflecting the graph of  $E(x, y)$  across  $y = 0$ .

Scaling:

3. The graph of  $E(\frac{x}{a}, y)$  is obtained by stretching the graph of  $E(x, y)$  horizontally by a factor of  $a$ .
4. The graph of  $E(x, \frac{y}{b})$  is obtained by stretching the graph of  $E(x, y)$  vertically by a factor of  $b$ .

Translations:

5. The graph of  $E(x - h, y)$  is obtained by shifting the graph of  $E(x, y)$   $h$  units right.
6. The graph of  $E(x, y - k)$  is obtained by shifting the graph of  $E(x, y)$   $k$  units up.

In 5. and 6., “right” and “up” are interpreted according the signs of  $h$  and  $k$ . For instance,  $-6$  units to the right means 6 units left.

In the special case that the  $E(x, y)$  has the form  $y = f(x)$ , then these rules tell us how changes to the function  $f(x)$  affect its graph.

Reflections:

- 1'. The graph of  $f(-x)$  is obtained by reflecting the graph of  $f(x)$  across  $x = 0$ .
- 2'. The graph of  $-f(x)$  is obtained by reflecting the graph of  $f(x)$  across  $y = 0$ .

Scaling:

- 3'. The graph of  $f(\frac{x}{a})$  is obtained by stretching the graph of  $f(x)$  horizontally by a factor of  $a$ .
- 4'. The graph of  $bf(x)$  is obtained by stretching the graph of  $f(x)$  vertically by a factor of  $b$ .

Translations:

- 5'. The graph of  $f(x - h)$  is obtained by shifting the graph of  $f(x)$   $h$  units right.
- 6'. The graph of  $f(x) + k$  is obtained by shifting the graph of  $f(x)$   $k$  units up.