MATH 111 FINAL REVIEW (Part 1) S. Agrest, Spring 2010

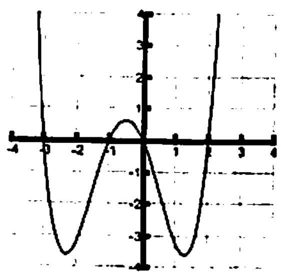
1. If $f(x)=\sqrt{x+2}$ and $g(x)=3x-1$, find and simplify
a) $(f\circ g)(x)$; b) $(g\circ f)(7)$;c) domain of $(f\circ g)(x)$ using interval notation .

2. Find the slope-intercept equation of the line passing through the point (2, 3) and parallel to the line y = -3x + 5.

3. Find the equation of the line passing through (-1, 3) and perpendicular to the line
2x + 3y = 9.

4. Solve the given inequalities, describe its solution set using interval notation.
a). $|3x+5|\le 2$
b). $\frac{2x+3}{x-1}\ge 1$
c). $x^3-2x^2-8x<0$
d). $\frac{2x}{x-3}\ge 5$
e). $x^2+3x-4>0$
f). $\frac{1}{2x}\le 5$

5. Use the graph of the function f given below to answer the following questions. Estimate your answer to one decimal place.



a) For what numbers x is f(x) = 0 ;
b) For what numbers x is f(x)< 0? (Use interval notation);
c) Find f(-2); and f(2
d) Over what interval(s) is f increasing (Use interval notation).

6. Determine the domain of each of the given functions.
a). f(x) $=\frac{2x-5}{3x^2-3}$
b). f(x) = $\sqrt{7x-10}$
c). f(x) = $3x^2+7x-2$

d). $f(x)=\sqrt{\frac{2x+3}{2x+1}}$ f. $\sin^{-1}x$ h. e^{x+2}

e). $f(x)=\sqrt[3]{x-1}$ g. $\ln(5-x)$ i. $\tan^{-1}(x-3)$

7. A new computer workstation costs $10,000. Its useful lifetime is 5 years, at which time it will be worth an estimated $2000. The company calculates its depreciation using the linear decline method .

a). Find the linear equation that expresses the value of the equipment as a function of time t.
b). How much will the equipment be worth after 2.5 years?

8. Find the x- and y-intercepts and sketch the graph of each of the quadratic function
a). $g(x)=x^2-4x+3$
b). $g(x)=-(x-1)^2$

9. Sketch the graph of each of the given functions

a). $f(x)=\begin{cases}-2x+1, if\_x<0\\ |x|, if\_x\geq 0\end{cases}$ b). $f(x)=\begin{cases}x+1, if\_x>1\\ x^3, if\_x\leq 1\end{cases}$

[redacted]

11. Express the area A of a circle as a function of its circumference C.

12. Find $(f\circ g)(x)$ and $(g\circ f)(x)$ for the given functions: f(x) = x + 2 and g(x) = 4-x^2.

13. Find the inverse functions of

a). $f(x)=\sqrt{2x-3}$; b). $f(x)=\frac{5}{2x+1}, x\neq -\frac{1}{2}$, y$\neq$0.

14. Sketch the graph of a rational function $R(x)$ that has the following properties:

a. $R(3)=0$

b. y-intercept at 6

c. $R(x)\to-\infty$ as $x\to 1^+$ and $R(x)\to\infty$ as $x\to 1^-$

d. $R(x)\to 2$ as $x\to\infty$ and $R(x)\to 2$ as $x\to-\infty$

15. Sketch the graph of polynomial P(x) that has zeros of multiplicity one at x = 0 and x = 1, has a zero of multiplicity three at x = -3, and satisfies P(x) $\to-\infty$ as x$\to-\infty$ and P(x) $\to\infty$ as x$\to\infty$.

16. The total revenue R (in thousand of dollars) for a certain product is given by $R = -4x^2 + 4x + 99$, where x is the number of units sold.

a). How many units should be produced for maximum revenue?

b). What is the maximum revenue?

17. Find all zeros, both real and complex

a). $f(x) = x^3 + x^2 - 15x + 18$

b). $f(x) = 3x^2 - 2x + 5$

c). $f(x) = x^3 - 3x^2 - 15x + 125$

[redacted]

19. For the given functions determine all vertical and horizontal asymptotes, x- and y-intercepts and sketch the graph.

a). $f(x) = \dfrac{2x+1}{x+1}$ b). $f(x) = \dfrac{x^2}{x^2 - 9}$ c). $f(x) = \dfrac{4x+12}{x^2 + 2x - 3}$

20. Find a polynomial with integer coefficients that has degree three and zeros 2 and 2i.

21. Given

x	f(x)	g(x)
0	4	4
1	2	3
2	0	2
3	1	1
4	4	0

Find a). $(f \circ g)(2)$; b). $(g \circ f)(2)$; c) $(f + g)(3)$.

22. A farmer with 2400 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

23. An object is thrown upward from the ground so that its height in feet after t seconds is given by $h(t) = -16t^2 + 96t$.

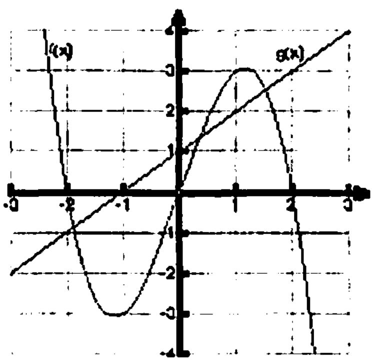
a). When does it reach its maximum height?

b). What is the maximum height?

24. The cubic polynomial P(x) has zeros at x = 1, x = 2, and x = -2 and y-intercept at 2. Determine P(x). Write your answer in standard form.

25. Sketch the graph of a rational function that satisfies all the following conditions:
a). $f(x) \to \infty \_ x \to 2^+, f(x) \to -\infty \_ x \to 2^-$
b). $f(x) \to \infty \_ x \to 0^+, f(x) \to -\infty \_ x \to 0^-$
c). Has a horizontal asymptote y = 0
d). f(1) = 0.

26. Find a polynomial of degree 4 with integer coefficients and a zero of multiplicity 2 at 1, and a zero at 2 + i.
27. Use the graphs of f and g in the figure to evaluate each expression



a). (f ∘ g)(1)
b). (g ∘ f)(-1)

28. The manager of a furniture factory finds that it costs $2200 to manufacture 100 chairs in one day and $4800 to manufacture 300 chairs in one day.
a). Assuming that the relationship between cost and the number of chairs produced is linear, find a function that expresses the cost of the chairs as a function of the number of chairs produced.
b). Using this function, find the factory's fixed cost (i.e. the cost incurred when the number of chairs produced is 0).

29. Find a polynomial with real coefficients of least degree that has zeros of multiplicity one at $x = 1$, $x = -2$, and $x = i$, and has a y-intercept $(0, 2)$. Write your answer in expanded form ($P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$).

30. A company is designing an open-top box with a square base that will hold 108 cubic centimeters of stuff.
a) Express the surface area $S(x)$ as a function of the length x of a side of the base.
[redacted]

31. Find the following limits.

a) $\lim\limits_{x\to 2}\dfrac{3-\sqrt{x+7}}{x-2}$; b) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}$; c). $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2-x-2}$.

32. Find the slope of the tangent line to the curve $f(x)=x^2+2$ at (1,3)[redacted]

33. Find the derivative of the function a) $f(x)=3x^2-2x$; b) $f(x)$ [redacted] $= 2\sqrt{x}$

34. Find an equation of the tangent line to the graph of $f(x)=\dfrac{2}{x}$ at x = 2.

35. Sketch the graph:

a) $f(x)=-(x-3)^{3/2}+1$; b) $f(x)=(3-x)^{2/3}-1$; c) $f(x)=-\sqrt{2x}+3$;

d) $y=\left|x^2+x-6\right|$; e) y [redacted] 2; c) $y=\sqrt[3]{x+1}-2$

36. Find the center and radius of the circle $x^2+y^2+2x-6y+7=0$.

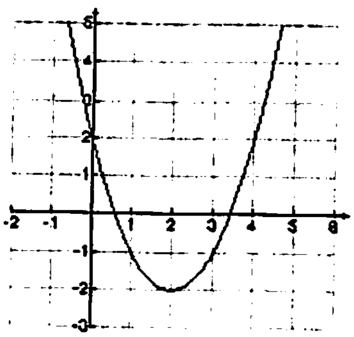
37. Given f(x)=$\frac{3}{x+1}$; g(x)=$\sqrt{2-x}$.

Find a). f∘g; b). g∘f and give the domain for each composition.

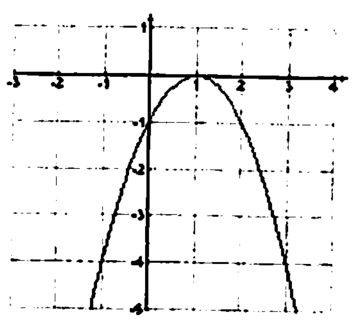
Answers

1. $a)\sqrt{3x+1}$; b) 8; c) $[-1/3,\infty)$
2. y = - 3x + 9
3. y = $\dfrac{3}{2}x+\dfrac{9}{2}$
4. a). [-7/3, -1]; b). (-∞,-4] U (1, +∞); c). $(-\infty,-2)\cup(0,4)$; d). (3, 5];
e). (-∞,-4) U (1, +∞); f). $(-\infty,0)\cup[\frac{1}{10},\infty)$.
5. a). x = -3, x = -1, x = 2, x = 0; b). (-3, -1) and (0, 2); c) f(-2) = -3; f(2) = 0;
d) $(-2.2,-.5)\cup(1.2,\infty)$.
6. a). (-∞, -1)U(-1, 1)U(1, +∞); b). [10/7, +∞), c). (-∞,+∞); f : [-1,1] g : (-∞, 5)
d). $[-\infty,-\frac{3}{2})\cup(-\frac{1}{2},\infty)$;e). (-∞,+∞) h : (-∞,∞) i : (-∞,∞)
7. a). y = -1600t + 10000; b). 6000.

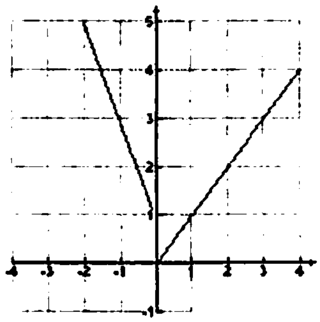
8. a). x-int: (1, 0), (3, 0)
y-int: (0, 3)



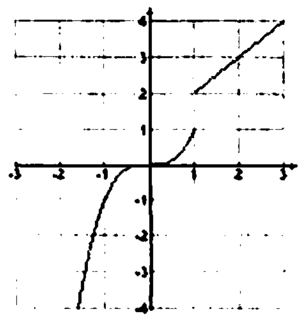
b). x-int: (1, 0),
y-int: (0, -1)



9. a).



b).



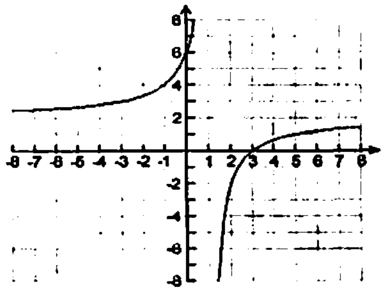
10. a). 2x + h - 4; b). 6x +3h; c) $-\frac{5}{x(x+h)}$; d) $\frac{2}{\sqrt{2x+2h+5}+\sqrt{2x+5}}$.

11. $A = \dfrac{C^2}{4\pi}$

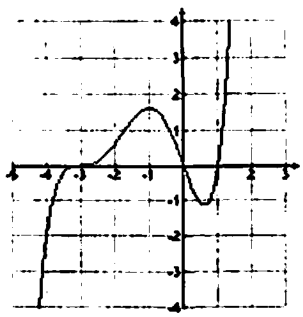
12. $(f \circ g)(x) = -x^2 + 6; (g \circ f)(x) = -x^2 - 4x$.

13. a). $f^{-1}(x) = \dfrac{x^2+3}{2}, x \geq 0$; b). $f^{-1}(x) = \dfrac{5-x}{2x}, x \neq 0$.

14.



15.

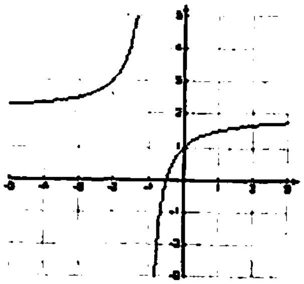


16. a). x = ½; b). R = 100.

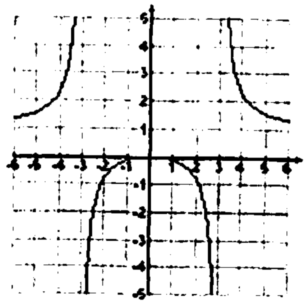
17. a). x = 2, x = 1.85, x = -4.85; b). $x = \frac{1}{3} \pm \frac{\sqrt{14}}{3}i$; c). x = -5, $x = 4 \pm 3i$.

18. min: (1, -7), max: (-2, 20), increasing: $(-\infty, -2) \cup (1, +\infty)$, decreasing: (-2, 1), f(x)>0 on (-3.3, 0) and $(1.9, \infty)$.

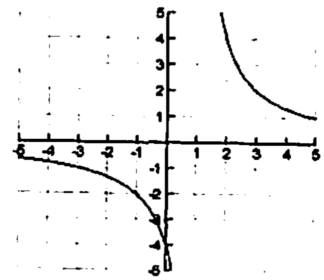
19. a). VA: x = -1
HA: y = 2
x-int: (-1/2, 0)
y-int: (0, 1)



b). VA: x = -3, x = 3
HA: y = 1
x-int: (0,0)
y-int: (0,0)



c). V.A x = 1; hole(-3,-1)
H.A: y = 0
x-int: none
y-int: (0, -4)



20. P(x) = $x^3 - 2x^2 + 4x - 8$.

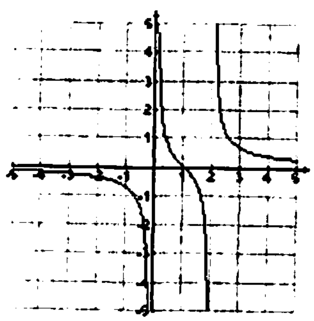
21. a). 0, b). 4, c). 2.

22. 720000 m^2.

23. a). 3; b). 144.

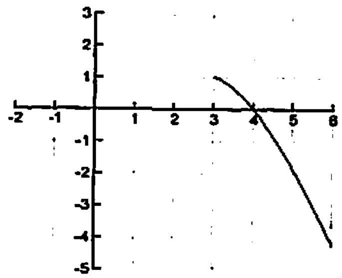
24. P(x) = $\frac{1}{2}x^3 - \frac{1}{2}x^2 - 2x + 2$.

25.

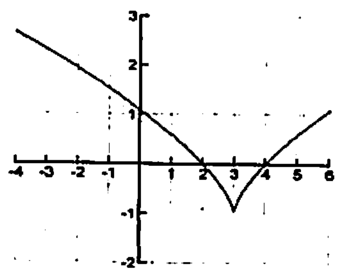


26. $x^4 - 6x^3 + 14x^2 - 14x + 5$
27. a). 0; b). –2.
28. a). C(x) =13x + 900; b). 900.
29. $-x^4 - x^3 + x^2 - x + 2$
30. a) $S(x) = x^2 + \frac{432}{x}$; b) x = 6cm.
31. a) –1/6; b) ½; c) 4/3.
32. 2
33. a) 6x -2, b) $\frac{1}{\sqrt{x}}$.
34. y = -1/2x +2.

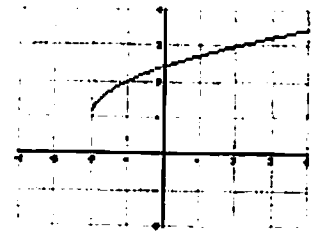
35. a)



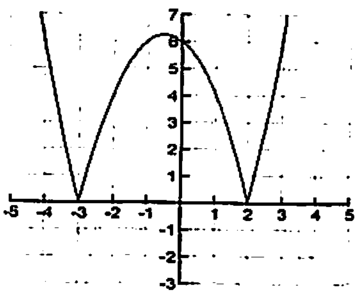
b)



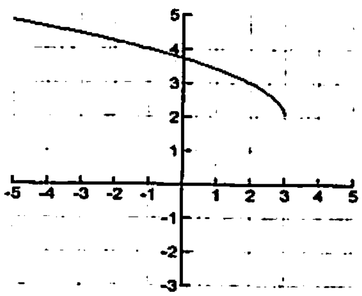
c)



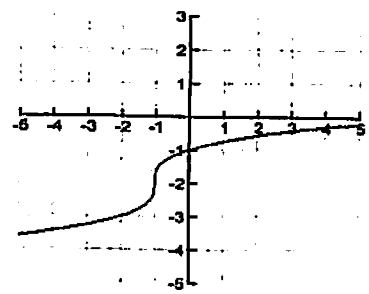
d)



e)



f)



36. center (-1, 3) , radius = $\sqrt{3}$

37. a). $\frac{3}{\sqrt{2-x}+1}$; D: $(-\infty,2]$; b). $\sqrt{\frac{2x-1}{x+1}}$; D: $(-\infty,-1)\cup[1/2,\infty)$;

MATH 111 FINAL REVIEW (Part 2) S. Agrest, Spring 2010

1. Find the value of each of the five remaining trig functions
a). sec x = 2, sin x < 0 b). tan x = ¾, sin x < 0.

2. Find the amplitude, period, Horizontal shift and sketch one period of each of the given

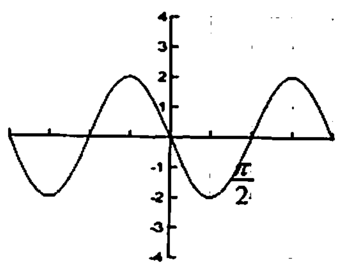
a). $f(x) = 3\sin(2x + \pi)$ b). $f(x) = \frac{1}{2}\cos(x - \frac{\pi}{3})$

c). $f(x) = \tan(x + \frac{\pi}{4})$

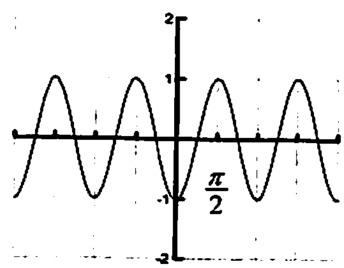
3. Find the exact value of

a). $\sin\left(\frac{7\pi}{6} - \frac{3\pi}{4}\right)$ b). $\cos\frac{\pi}{12}$ c). $\cos(\frac{\pi}{6} + \frac{\pi}{4})$

4. Find an equation of the cosine function whose graph matches the given curve
a)



b)



5. If sin a = 3/5, $0 < a < \frac{\pi}{2}$ and cos b = $\frac{2}{\sqrt{5}}, -\frac{\pi}{2} < b < 0$, find

a). sin (a +b) b). sin 2a

6. Solve each equation on the interval $0 \le x < 2\pi$
a). cos x = ½ b). sin 2x = -1
c). $\sin^2 x + \sin x = 0$ d). $2\cos^2 x - 3\cos x + 1 = 0$
e). $\cos 3x = \frac{\sqrt{2}}{2}$; f). tan x = 1; g) $1 + \sin x = 2\cos^2 x$; h) cos(2x) = cosx;
i) $2\sin^2 x + \sin x - 1 = 0$;

7. Find the exact value of each of the given

a) $\sin(\arctan\frac{3}{4})$; b) cos(arcsin 5/13); c) $\cos^{-1}(-\frac{\sqrt{2}}{2})$; d). $\sin^{-1}(\frac{1}{2})$; e) $\sin(\sin^{-1}\frac{2}{3})$;

f) $\cos(\arctan\frac{12}{5} + \arcsin\frac{3}{5})$; g) arcsec(-2); h) cos(arccos -.2); i) sin(arcsin 3); j)

$\sin^{-1}(\sin(\frac{\pi}{7}))$, k) $\tan^{-1}(-\frac{\sqrt{3}}{3})$.

8. Use the given information to find the remaining sides and angles of the triangles
a) given: $A = 150^\circ$, $C = 20^\circ$, a = 200.

b) given A = 30^0, b = 10, c = 20.

9. Graph

a). $f(x) = \ln(x-1)-1$ b). $f(x) = e^{(x+3)} + 2$

10. Evaluate each expression without using a calculator

a). $\log_2 32$ b). $\ln \frac{1}{e}$ c). $e^{4\ln 2}$

11. Solve for x:

a). $e^{3x} = 12$ b). $3^{2x} = e^{5x-1}$ c). $\ln(x-2) - \ln(2x+1) = 2$

d). $3^{2t} - 3^t - 6 = 0$ e). $\log_2(5+x) = 3$ f) $2\ln(x+1) = \ln(1-2x)$

12. The half-life of radium is 1690 years. If there are initially 10 grams present, how much will be present in 50 years?

[redacted]

14. Find the exact value

a). $\csc \frac{8\pi}{3}$; b). $\sin(\frac{4\pi}{3})$; c). $\cos(\frac{5\pi}{4})$; d). $\tan(\frac{5\pi}{2})$

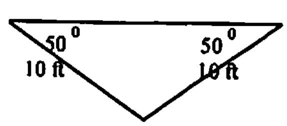
15.Find the inverse function

a). $f(x) = e^{5-2x}$; b).$f(x) = \ln(7x-9)$

16. Use the properties of logarithms to simplify the expression so that the result does not contain logarithms of products, quotients, or powers.

a). $\log(x^3(x+1)^{1/2})$ b). $\ln\left(\frac{s^3\sqrt{t}}{(t^2+1)^4}\right)$

17. In preparation for an outdoor rock concert, a stage crew must determine how far apart to place the two large speaker columns on stage. What generally works best is to place them at 50^0 angles to the center of the front row. The distance from the center of the front row to each of the speakers is 10 ft. How far apart does the crew need to place the speakers on stage? Round your answer to one decimal place.



18. If $\sec x = \frac{3}{2}$ and $\tan x < 0$, find $\sin(2x)$.

[redacted]

20. A culture starts with 10,000 bacteria, and the number doubles every 40 minutes.
a) Find a formula for the number of bacteria at time t in min.
b) After how many minutes will there be 50,000 bacteria?

21. Polonium-210 (^{210}Po) has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg.
a) Find a formula for the amount of the sample remaining after t days.
b) Find the mass remaining after 1 year.

[redacted]

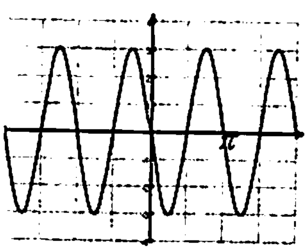
23. If the sun is 30^0 above the horizon, find the length of a shadow cast by a silo that is 70 feet high.

24. The angles of elevation to an airplane from two points A and B on level ground are 51^0 and 68^0, respectively. The points A and B are 6 mi apart, and the airplane is between these positions in the same vertical plane. Find the altitude of the airplane.

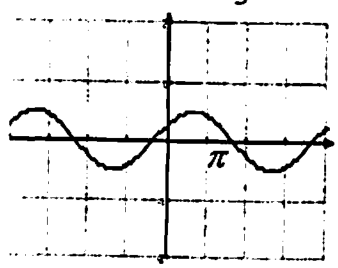
Answers:

1. a). $\sin x = -\frac{\sqrt{3}}{2}$, $\cos x = ½$, $\tan x = -\sqrt{3}$, $\cot x = -\frac{\sqrt{3}}{3}$, $\csc x = \frac{-2\sqrt{3}}{3}$;
b). $\sin x = -3/5$, $\cos x = -4/5$, $\cot x = 4/3$, $\sec x = -5/4$, $\csc x = -5/3$.

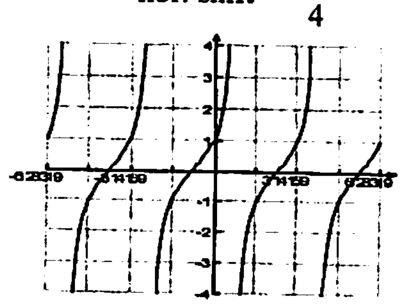
2. a). ampl. = 3
period = π
hor. shift = $-\frac{\pi}{2}$



b). ampl. = ½
period = 2π
hor. shift = $\frac{\pi}{3}$



c). ampl. = none
period = π
hor. shift = $-\frac{\pi}{4}$



3. a). $\frac{1}{4}(\sqrt{2}+\sqrt{6})$; b). $\frac{1}{4}(\sqrt{2}+\sqrt{6})$; c). $\frac{1}{4}(\sqrt{6}-\sqrt{2})$.

4. a) $y = 2\cos(2x + \frac{\pi}{2})$; b). $y = \cos(2x - \pi)$

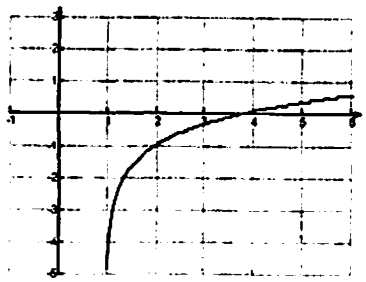
5. a). $\frac{2\sqrt{5}}{25}$, b). $\frac{24}{25}$.

6. a). x = $\frac{\pi}{3}, \frac{5\pi}{3}$.; b). x = $\frac{3\pi}{4}, \frac{7\pi}{4}$;c). x = 0, $\pi, \frac{3\pi}{2}$; d). $x = 0, \frac{\pi}{3}, \frac{5\pi}{3}$.;
e). x = $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$; f). x = $\frac{\pi}{4}, \frac{5\pi}{4}$; g) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$, h) $0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$;
i) $x = \pi/6, 5\pi/6, 3\pi/2$

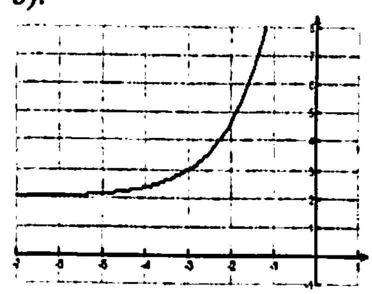
7. a). 3/5; b). 12/13; c). $\frac{3\pi}{4}$; d). $\frac{\pi}{6}$; e). 2/3; f) -16/65, g) $\frac{2\pi}{3}$; h) -.2; i) dne; j) $\frac{\pi}{7}$, k) $\frac{\pi}{6}$.

8. a). B = 10°, b = ~~69.46~~, c = ~~136.81~~ see below; b). ~~B = 23.8°, C = 126.2°, a = 12.4.~~ see below

9. a).



b).



10. a). 5, b). -1, c).16.

11. a). x = $\frac{\ln 12}{3}$, b). x = $\frac{-1}{2\ln 3 - 5}$ c). x = $\frac{e^2+2}{1-2e^2}$; d). t = 1; e). x = 3; f) x = 0

12. ~~9.797.~~ $10 \cdot (1/2)^{5/169}$

~~13. $12.46.~~

14. a). $\frac{2\sqrt{3}}{3}$; b). $-\frac{\sqrt{3}}{2}$; c). $-\frac{\sqrt{2}}{2}$; d). dne.

15. a). $y = \frac{5 - \ln x}{2}$; b). $y = \frac{e^x + 9}{7}$.

16. a). 3 log x + ½ log(x+1); b). 3ln s + ½ ln t – 4 ln(t^2+1) .

17. ~~12.9 ft~~ $10 \cdot \frac{\sin 80^{\circ}}{\sin 50^{\circ}}$

18. $\frac{-4\sqrt{5}}{9}$

~~19. 7.4 years~~

20. a) $Q = 10{,}000e^{\frac{\ln 2}{40}t}$; b) ~~92.9.~~ $(40 \ln 5)/\ln 2$

21. a) $Q = 300e^{-\frac{\ln 2}{140}t}$; b) ~~49.256.~~ $300\, e^{-\frac{\ln 2}{140}}$ (also = $300\left(\frac{1}{2}\right)^{\frac{1}{140}}$)

23. ~~121.2.~~ $70\sqrt{3}$

24. ~~4.94.~~ $6/(\tan 39^{\circ} + \tan 21^{\circ})$

8a) $b = \frac{200}{\sin 150^{\circ}} \cdot \sin 10^{\circ}$ $c = \frac{200}{\sin 150^{\circ}} \cdot \sin 20^{\circ}$

8b. $a = \sqrt{500 - 400 \cos 30^{\circ}} = \sqrt{500 - 200\sqrt{3}}$

$c = \sin^{-1}\left(\frac{10}{\sqrt{500-200\sqrt{3}}}\right)$

$B = 150 - \sin^{-1}\left(\frac{10}{\sqrt{500-200\sqrt{3}}}\right)$.

More Final Review Problems for MATH 111

(See http://kunklet.people.cofc.edu/More111Probs.html for more practice problems.)

1. Graph the function by hand. Use a graphing calculator to check your answers.

a. $y = (1-x)^{3/4}$ b. $y = -(x-1)^{9/7}$ c. $y = (4+x)^{-10/7} - 6$

2. Find $f^{-1}(x)$ and the domain and range of f^{-1}.

a. $f(x) = 1 - \dfrac{1}{x+7}$ b. $f(x) = x^3 + 2$ c. $f(x) = 3\ln(1+e^x)$

3. Find the derivative of each function $f(x)$ below.

a. $f(x) = x^3 + 3x,\ a = -2$ b. $f(x) = \dfrac{1}{2x+3},\ a = 0$ c. $f(x) = \sqrt{2x+1},\ a = 4$

4. Make a rough sketch of the following functions **by hand**. Check your work by graphing the functions on a graphing calculator.

a. $2x(x-1)^3(x-3)^3(x+4)^3(x-5)^2$ b. $-2(x+4)^2(x+3)(x+2)(x^2-1)$

5. Find all asymptotes for the graphs of the following rational functions.

a. $r(x) = \dfrac{3-x}{8x^2-6x+1}$ b. $r(x) = \dfrac{3x^2+5x+2}{x^2-3x-4}$ c. $r(x) = \dfrac{2x^2-1}{x+2}$

6. A photographer climbs up a tree in order to see Michelle Obama. If the camera is 20m above the ground and the angle of depression of the camera is 22°, how far is the photographer from the first lady?

7. See Problem 6. Secret service agents standing beside Michelle Obama run to the photographer's tree and shake it vigorously. How far did they have to run?

8. What is the angle of elevation of the sun if a 6ft man casts a 10.5ft shadow?

9. Two ships leave a harbor at the same time, each traveling in a straight line. One ship travels 4 km/hr, while the other ship travels 3 km/hr. If the angle between their courses is 54°, find the distance between them after 2 hours.

Answers

2a. $f^{-1}(x) = \frac{7x-6}{1-x}$; dom $= (-\infty, 1) \cup (1, \infty)$, range $= (-\infty, -7) \cup (-7, \infty)$. 2b. $f^{-1}(x) = \sqrt[3]{x-2}$; dom = range = (∞, ∞). 2c. $f^{-1}(x) = \ln(e^{x/3} - 1); D = (0, \infty); R = (-\infty, \infty)$ 3a. $f'(x) = 3x^2 + 3$. 3b. $f'(x) = \frac{-2}{(2x+3)^2}$. 3c. $f'(x) = \frac{1}{\sqrt{2x+1}}$. 5a. $y = 0, x = 1/2, x = 1/4$ 5b. $y = 3, x = 4$ (graph has a hole at the point $(-1, 1/5)$.) 5c. $y = 2x - 4, x = -2$ 6. $20\csc 22°$ (=53.389m) 7. $20\cot 22°$ (= 49.501m) 8. $\tan^{-1}(6/10.5)$ (=29.744°) 9. $\sqrt{8^2 + 6^2 - 96\cos 54°}$