1 (10 pts). These 3 problems refer to the figure at the right.

a. Find \( b \) if \( a = 2, \beta = 110^\circ \) and \( \gamma = 20^\circ \).
b. Find \( c \) if \( a = 1, b = 2 \) and \( \gamma = 10^\circ \).
c. Find \( \alpha \) if \( a = 5, b = 6 \) and \( c = 2 \).

**Solution:**

1a. (Source: 4.12.more.1d) This problem is classified as AAS (given two angles and one side). First find \( \alpha = 180^\circ - \beta - \gamma = 50^\circ \). Then, by the Law of Sines,

\[
\frac{b}{\sin \beta} = \frac{a}{\sin \alpha} = \frac{2}{\sin 50^\circ}.
\]

\[
b = \frac{2 \sin 110^\circ}{\sin 50^\circ}.
\]

1b. (Source: 4.12.more.1) This problem is classified as SAS (given two sides and the angle between them). By the Law of Cosines,

\[
c^2 = a^2 + b^2 - 2ab \cos \gamma = 1 + 4 - 4 \cos 10^\circ \]

\[
c = \sqrt{5 - 4 \cos 10^\circ}.
\]

1c. (Source: 4.13.more.2) This problem is classified as SSS (given three sides). To find \( \alpha \), first find its cosine. By the Law of Cosines,

\[
a^2 = b^2 + c^2 - 2bc \cos \alpha \]

\[
25 = 36 + 4 - 24 \cos \alpha \]

\[
\cos \alpha = \frac{15}{24} = \frac{5}{8}.
\]

As an angle inside a triangle, \( \alpha \) must lie in \([0, \pi]\), which is exactly the range of the inverse cosine. Therefore, \( \alpha = \cos^{-1} \left( \frac{5}{8} \right) \).