1. Find $f^{-1}(x)$ if $f(x) = \frac{3x + 1}{2x - 5}$. State the domain and range of $f$ and of $f^{-1}$ in interval notation.

**Solution:**

1. (Source: 2.8.more.1n) Set $y = f(x)$ and solve for $x$. (At each stage in doing so, we find a different equation for the same curve.)

\[
y = \frac{3x + 1}{2x - 5}
\]

\[
(2x - 5)y = 3x + 1
\]

\[
2xy - 5y = 3x + 1
\]

\[
2xy - 3x = 5y + 1
\]

\[
(2y - 3)x = 5y + 1
\]

\[
x = \frac{5y + 1}{2y - 3} = f^{-1}(y)
\]

So,

\[
f^{-1}(x) = \frac{5x + 1}{2x - 3}.
\]

The only restriction that $f^{-1}(x)$ places on $x$ is that it can’t be $3/2$, since this causes a zero in the denominator:

\[
\text{domain } f^{-1} = (-\infty, 3/2) \cup (3/2, \infty) = \text{range } f.
\]

Similarly, the only $x$ not in the domain of $f$ is $5/2$:

\[
\text{domain } f = (-\infty, 5/2) \cup (5/2, \infty) = \text{range } f^{-1}
\]