MATH 111–05 (Kunkle), Quiz 4 — Name: ____________________

10 pts, 10 minutes — Feb. 13, 2018

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1 (10 pts). Find $f^{-1}(x)$ if $f(x) = \dfrac{3x+1}{2x-5}$. State the domain and range of f and of f^{-1} in interval notation.

Solution:

1. (Source: 2.8.more.1n) Set $y = f(x)$ and solve for x. (At each stage in doing so, we find a different equation for the same curve.)

$$\begin{aligned} y &= \frac{3x+1}{2x-5} \\ (2x-5)y &= 3x+1 \\ 2xy - 5y &= 3x+1 \\ 2xy - 3x &= 5y+1 \\ (2y-3)x &= 5y+1 \\ x &= \frac{5y+1}{2y-3} = f^{-1}(y) \end{aligned}$$

So,

$$f^{-1}(x) = \frac{5x+1}{2x-3}.$$

The only restriction that $f^{-1}(x)$ places on x is that it can't be $3/2$, since this causes a zero in the denominator:

$$\text{domain}\, f^{-1} = (-\infty, 3/2) \cup (3/2, \infty) = \text{range}\, f.$$

Similarly, the only x not in the domain of f is $5/2$:

$$\text{domain}\, f = (-\infty, 5/2) \cup (5/2, \infty) = \text{range}\, f^{-1}$$