1 (10 pts). Sketch the graphs of the following functions. Include all asymptotes and their equations. Find all intercepts. Label your answers so I can tell your $x$-intercepts from your $y$-intercepts.

\begin{itemize}
\item[a.] $f(x) = x^{-2/5}$
\item[b.] $g(x) = 3 + (x - 2)^{-2/5}$.
\end{itemize}

(Source: 2.2.more.1f)

1a. In the first quadrant, the graph of $y = x^{-2/5} = \frac{1}{(\sqrt[5]{x})^2}$ looks roughly like $y = x^{-1}$. The axes $y = 0$ and $x = 0$ are asymptotes because $|y|$ is huge when $|x|$ is tiny and tiny when $|x|$ is huge. $\sqrt[5]{x}$ is defined for negative $x$-values, and when we compute its square and take reciprocals, the result is positive, so the graph of $y = x^{-2/5}$ looks the graph on the left:

\begin{center}
\begin{tabular}{c}
\includegraphics[width=0.4\textwidth]{graph1a}
\end{tabular}
\end{center}

1b. To obtain the graph of $y = 3 + (x - 2)^{-2/5}$, shift the graph up 3 and to the right 2. This makes the two new asymptotes $y = 3$ and $x = 2$. See the graph on the right.

As our drawing suggests, the graph has no $x$-intercept. This because when we set $y = 0$ and solve for $x$, we come to the equation

\[ -3 = (x - 2)^{-2/5} = \left(\frac{1}{\sqrt[5]{x - 2}}\right)^2. \]

Since the right side is a square, this has no real solutions.

At $x = 0$, we calculate the $y$-intercept to be $y = 3 + (0 - 2)^{-2/5}$. Since $(-1)^{-2/5} = 1$, the $y$-intercept can be more simply written as either $3 + 2^{-2/5}$ or $3 + \frac{1}{\sqrt[5]{4}}$. 

\[ \text{(Source: 2.2.more.1f)} \]