

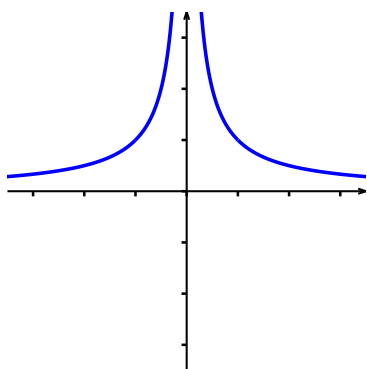
1 (10 pts). Sketch the graphs of the following functions. Include all asymptotes and their equations. Find all intercepts. Label your answers so I can tell your  $x$ -intercepts from your  $y$ -intercepts.

a.  $f(x) = x^{-2/5}$

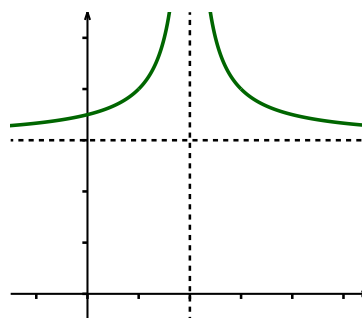
b.  $g(x) = 3 + (x - 2)^{-2/5}$ .

(Source: 2.2.more.1f)

1a. In the first quadrant, the graph of  $y = x^{-2/5} = \frac{1}{(\sqrt[5]{x})^2}$  looks roughly like  $y = x^{-1}$ . The axes  $y = 0$  and  $x = 0$  are asymptotes because  $|y|$  is huge when  $|x|$  is tiny and tiny when  $|x|$  is huge.  $\sqrt[5]{x}$  is defined for negative  $x$ -values, and when we compute its square and take reciprocals, the result is positive, so the graph of  $y = x^{-2/5}$  looks the graph on the left:



a.  $y = x^{-2/5}$



b.  $y = 3 + (x - 2)^{-2/5}$

1b. To obtain the graph of  $y = 3 + (x - 2)^{-2/5}$ , shift the graph up 3 and to the right 2. This makes the two new asymptotes  $y = 3$  and  $x = 2$ . See the graph on the right.

As our drawing suggests, the graph has no  $x$ -intercept. This because when we set  $y = 0$  and solve for  $x$ , we come to the equation

$$-3 = (x - 2)^{-2/5} = \left( \frac{1}{\sqrt[5]{x - 2}} \right)^2.$$

Since the right side is a square, this has no real solutions.

At  $x = 0$ , we calculate the  $y$ -intercept to be  $y = 3 + (0 - 2)^{-2/5}$ . Since  $(-1)^{-2/5} = 1$ , the  $y$ -intercept can be more simply written as either  $3 + 2^{-2/5}$  or  $3 + \frac{1}{\sqrt[5]{4}}$ .