No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

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1. Let $s(x) = 3x^2 - 12x + 17$. Work in the space below and label your answers to the following questions.

a(5 pts). Rewrite $s(x)$ in standard form.
b(6 pts). Find the coordinates of the vertex and all intercepts of the graph of $s(x)$.
c(4 pts). Sketch the graph of $s(x)$ on the axes below. Your drawing needn't to be to scale but should show the points you found in part b.
d(3 pts). Give the maximum and minimum values of $s(x)$, or state that they do not exist.

2(24 pts). Write the solution set in interval form.

a. $|4x - 3| \geq 15$ b. $|3x + 4| < 9$ c. $\dfrac{2}{x+3} - \dfrac{1}{x+1} \leq 0$

3(4 pts). Find the domain of the function $\sqrt{3 - 2x}$. Give your answer in interval form.

4(20 pts). Let $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{x+1}{x}$. Find the following functions and their domains. Express all functions as simple fractions in lowest terms. No fractions-within-fractions. State domains in interval notation.

a. $\big(f \div g\big)(x)$ b. $\big(f \circ g\big)(x)$

5(4 pts). Solve for y in the equation $(y - x + 1)(y - x^2) = 0$.
Express your answer in the form "$y =$ (one function of x) or $y =$ (another function of x)."

6(13 pts). Sketch the graphs of the following functions. Your drawings should clearly show how the two graphs are related. Find all intercepts and the equations of all asymptotes (if any), and include these in your drawing.

a. $p(x) = x^{3/5}$ b. $q(x) = -(x+1)^{3/5} - 8$

7(5 pts). Sketch the graph of the function $r(x) = |3x + 4|$. Find all intercepts and include them in your drawing.

8(12 pts). Find an equation of the curve described in each part.

a. The line through the points $(1, 5)$ and $(-2, 9)$.

b. The line through the points $(1, 5)$ and $(1, 9)$.

c. The circle having a diameter with endpoints $(1, 5)$ and $(-1, 9)$.

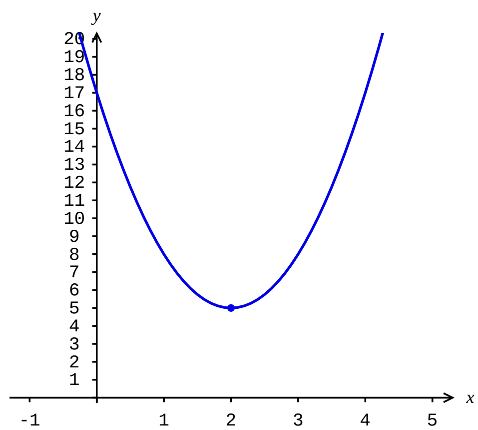
1a.(Source: 2.4.19-20,more.1i) Complete the square:

$$s(x) = 3x^2 - 12x + 17 = 3(x^2 - 4x) + 17 = 3(x^2 - 4x + 4) + 17 - 12 = 3(x-2)^2 + 5$$

1b. The vertex is at $(2, 5)$. To find the y-intercept, set $x = 0$ and calculate $y = 17$. When we set $y = 0$, the result is

$$0 = 3(x-2)^2 + 5 \Rightarrow -5 = 3(x-2)^2,$$

which has no real solutions. Therefore, the graph has no x-intercepts.



1c. See drawing at right.

1d. The minimum value of $s(x)$ is its value at the vertex, or 5. It has no maximum value.

2a.(Source: 1.2.more.2e) First rewrite the problem in equivalent form without absolute values. To do this, see Theorem 1.2.2, p. 13 of the text. $|4x - 3| \geq 15$ means

$$\begin{array}{rcr} 4x - 3 \leq -15 & \text{or} & 4x - 3 \geq 15 \\ 4x \leq -12 & & 4x \leq 18 \\ x \leq -3 & \text{or} & x \leq \frac{18}{4} = \frac{9}{2} \end{array}$$

The solution set is $(-\infty, -3] \cup [\frac{9}{2}, \infty)$.

2b.(Source: 1.2.more.2d) $|3x + 4| < 9$ means

$$-9 < 3x + 4 < 9 \quad \Rightarrow \quad -13 < 3x < 5 \quad \Rightarrow \quad -\frac{13}{3} < x < \frac{5}{3}$$

The solution set is $(-\frac{13}{3}, \frac{5}{3})$.

2c.(Source: 1.1.57) Add the fractions.

$$\frac{(x+1)}{(x+1)} \cdot \frac{2}{(x+3)} - \frac{1}{x+1} \cdot \frac{(x+3)}{(x+3)} = \frac{2(x+1) - (x+3)}{(x+1)(x+3)} = \frac{x-1}{(x+1)(x+3)} \leq 0$$

Now make a sign chart.

$$\begin{array}{rl} x - 1: & - - - - - - - - - - - - - - - - 0 + + + + + + + \\ x + 1: & - - - - - - - - - - - - - 0 + + + + + + + + + + + + \\ x + 3: & - - - - - - 0 + + + + + + + + + + + + + + + + + \\ \frac{x-1}{(x+1)(x+3)}: & - - - - - \text{DNE} + + + \text{DNE} - - - - 0 + + + + + + \\ \hline x: & \qquad\qquad -3 \qquad\qquad -1 \qquad\qquad 1 \end{array}$$

The solution set is $(-\infty, -3) \cup (-1, 1]$.

3.(Source: 2.1.more.1b) We only need $3 - 2x \geq 0$ for the square root to be defined. Solving gives $\frac{3}{2} \geq x$. In interval form, the domain is $(\infty, \frac{3}{2}]$.

4a.(Source: 2.6.6,16) $\big(f \div g\big)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x-2}}{\frac{x+1}{x}}$. The three fractions place three restrictions on x:

$$x - 2 \neq 0 \qquad x + 1 \neq 0 \qquad \frac{x}{x+1} \neq 0$$

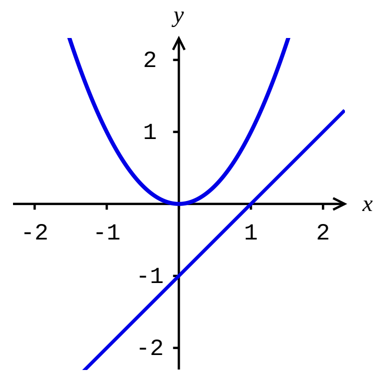
The domain is all x values except 2, -1, and 0. In interval form, $(\infty, -1) \cup (-1, 0) \cup (0, \infty)$. Now go ahead and simplify: $\frac{1}{x-2} \cdot \frac{x}{x+1} = \frac{x}{(x-2)(x+1)}$.

4b. $\big(f \circ g\big)(x) = f(g(x)) = f\left(\frac{x+1}{x}\right) = \frac{1}{\frac{x+1}{x} - 2}$. The little fraction requires that $x \neq 0$, and the big fraction requires $\frac{x+1}{x} - 2 \neq 0$. Multiply both sides by x to obtain $x + 1 - 2x \neq 0$, or $x \neq 1$. The domain is $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.
The easiest way to simplify $f \circ g$ is to multiply top and bottom of the big fraction by x:

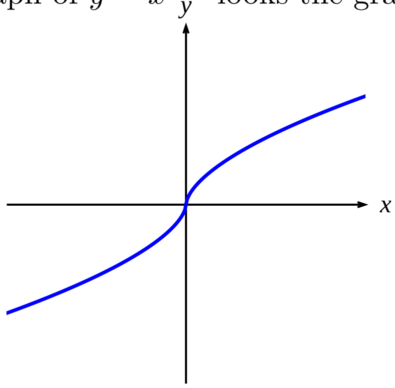
$$\frac{1}{\frac{x+1}{x} - 2} \cdot \frac{x}{x} = \frac{x}{x + 1 - 2x} = \frac{x}{1 - x}.$$

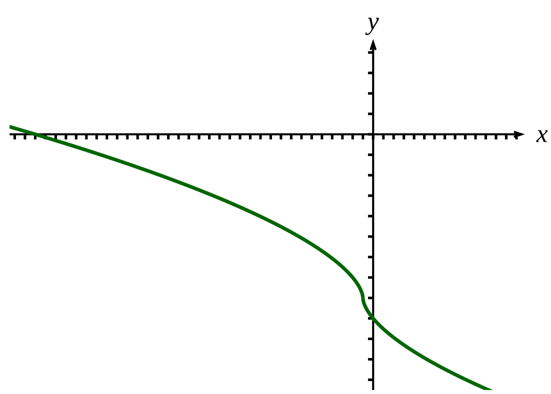
5.(Source: 2.7.7) $(y - x + 1)(y - x^2) = 0$ if and only if
$y - x + 1 = 0$ or $y - x^2 = 0$ if and only if
$y = x - 1$ or $y = x^2$. **(done)**

No graph is required, but this tells us that the graph of $(y - x + 1)(y - x^2) = 0$ consists of the line $y = x - 1$ and the parabola $y = x^2$. See the figure at right.



6a.(Source: 2.2.more.1u) Because $0 < \frac{3}{5} < 1$, the graph of $y = x^{3/5}$ looks in the first quadrant roughly like $y = x^{1/2}$. It has no asymptotes and its only intercept is the origin. $\sqrt[5]{x}$ is defined for negative x-values, and when we compute its cube the result is negative, so the graph of $y = x^{3/5}$ looks the graph on the left:





a. $y = x^{3/5}$ b. $y = -(x+1)^{3/5} - 8$

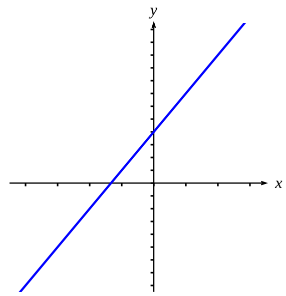
6b. To obtain the graph of $y = -(x+1)^{3/5} - 8$, reflect the graph across the x-axis, then shift the graph down 8 and to the left 1. See the graph on the right.

The curve has no asymptotes. To find the x-intercept, set $y = 0$ and solve for x:

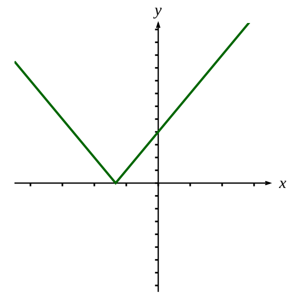
$$(x+1)^{3/5} = -8 \quad \Rightarrow \quad x+1 = (-8)^{5/3} = (\sqrt[3]{-8})^5 = (-2)^5 = -32 \quad \Rightarrow \quad x = -33.$$

To find the y-intercept, set $x = 0$ and calculate the $y = -(1)^{3/5} - 8 = -1 - 8 = -9$.

7.(Source: 2.5.24) First graph the line $y = 3x + 4$. Its intercepts are $(-\frac{4}{3}, 0)$ and $(0, 4)$. See the graph on the left. To obtain the graph of $y = |3x+4|$, reflect across the x-axis any parts of the graph that below the axis. See graph above on the right.



$y = 3x + 4$



$y = |3x + 4|$

8a.(Source: 1.4.19, 2.3.23,27) The line through $(1, 5)$ and $(-2, 9)$ has slope $\frac{9-5}{-2-1} = -\frac{4}{3}$. In point-slope form, the line is $y - 5 = -\frac{4}{3}(x - 1)$.

8b. This vertical line has no slope. Its equation is $x = 1$.

8c. The center of the circle is at the midpoint of the diameter, $(\frac{1+(-1)}{2}, \frac{5+9}{2}) = (0, 7)$. Its radius is the distance from the center to either endpoint of the diamater, $\sqrt{1^2 + 2^2} = \sqrt{5}$, and so the equation of the circle is $x^2 + (y - 7)^2 = 5$.