No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(4 pts). Express as a fraction in lowest terms, if it exists.

a. $\frac{7}{8} - \frac{17}{12}$ b. $\frac{8}{13} + 2$ c. $\frac{6}{35} \times \frac{5}{12}$ d. $\frac{0}{2} \div \frac{2}{3}$

2(5 pts). Rationalize the denominator and simplify the result:

a. $\frac{3}{2\sqrt{6}}$ b. $\frac{11}{5+\sqrt{3}}$

3(6 pts). Rewrite the expression in simplest radical form or state that it does not exist.

a. $\sqrt{-36}$ b. $\sqrt[5]{-96}$ c. $16^{-3/2}$

4(9 pts). Simplify. Write your answer to part b. without negative exponents. Write your answer to part c. without fractions (except possibly in an exponent).

a. $a^8 b^{-2}(4b)^{-3}(2ab^2)^3$ b. $xy(x^{1/2}y^{-1/3})^{12}$ c. $\dfrac{\frac{9a}{b} \cdot \frac{b^3}{a^4}}{\frac{3}{ab}}$

5(9 pts). Factor the polynomial completely over the real numbers.

a. $x^2y^5 - 8y^2x^5$ b. $2x^2 + x - 28$ c. $2x^3 - 3x^2 - 18x + 27$

6(15 pts). Perform the indicated operation and simplify the resulting rational function.

a. $\dfrac{1}{x-2} + \dfrac{1}{x+2}$ b. $\dfrac{x-3}{2x+1} \cdot \dfrac{x+3}{x^2-4x+3}$

c. $\dfrac{x^2+4}{x^2-9} \div \dfrac{x+2}{x+3}$ d. $\dfrac{2}{3x^2-23x-8} - \dfrac{1}{3x^2+13x+4}$

1.(Source: BootCamp.3e, 4f, 5g, 6f) Tips: a. to add fractions, it's best to use the *smallest* common denominator. c. You don't need a common denominator to multiply, and it's best to look for common factors *before* multiplying. d. $0 \div 4$ is well-defined and equals 0.
a. $\frac{7}{8} - \frac{17}{12} = \frac{3}{3} \cdot \frac{7}{8} - \frac{17}{12} \cdot \frac{2}{2} = \frac{21-34}{24} = -\frac{13}{24}$. b. $\frac{8}{13} + \frac{2}{1} = \frac{8}{13} + \frac{2}{1} \cdot 1313 = \frac{8+26}{13} = \frac{34}{13}$.
c. $\frac{6}{5\cdot 7} \times \frac{5}{2\cdot 6} = \frac{1}{7\cdot 2} = \frac{1}{14}$. d. $\frac{0}{2} \div \frac{2}{3} = 0 \div \frac{2}{3} = 0$.

2.(Source: BootCamp.7m, 8o) Tips: a. In order to rationalize the denominator, it is not necessary to multiply top and bottom by 2. This only creates a common factor that must be cancelled in the next step.
a. $\frac{3}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{2\cdot 6} = \frac{\sqrt{6}}{4}$. b. $\frac{11}{(5+\sqrt{3})} \cdot \frac{(5-\sqrt{3})}{(5-\sqrt{3})} = \frac{11(5-\sqrt{3})}{5^2-\sqrt{3}^2} = \frac{11(5-\sqrt{3})}{25-3} = \frac{11(5-\sqrt{3})}{22} = \frac{5-\sqrt{3}}{2}$.

3.(Source: BootCamp.9d, 10k, 11n)
a. There is no real number whose square equals -36, so $\sqrt{-36}$ does not exist. (I would also give full credit for the complex number $6i$.)
b. Factor $96 = 3 \cdot 32 = 3 \cdot 2^5$. Then $\sqrt[5]{-96} = \sqrt[5]{-32 \cdot 3} = \sqrt[5]{-1}\,\sqrt[5]{32}\,\sqrt[5]{3} = -2\sqrt[5]{3}$.
c. $16^{-3/2} = \frac{1}{\sqrt{16}^3} = \frac{1}{4^3} = \frac{1}{64}$.
Tip: $(\sqrt{16})^3 = \sqrt{(16^3)}$, but the second is harder to calculate.

4.(Source: BootCamp.12bhr)
a. $a^8b^{-2}4^{-3}b^{-3}2^3a^3(b^2)^3 = a^8b^{-2}\frac{1}{4^3}b^{-3}2^3a^3b^6$. Simplify $\frac{2^3}{4^3}$ to $(\frac{1}{2})^3 = \frac{1}{8}$ and the answer becomes $\frac{1}{8}a^{8+3}b^{-2-3+6} = \frac{1}{8}a^{11}b$.
b. $xy(x^{1/2}y^{-1/3})^{12} = xyx^6y^{-4} = x^7y^{-3} = \frac{x^7}{y^3}$.
c. $\frac{9a}{b} \cdot \frac{b^3}{a^4} \cdot \frac{ab}{3} = \frac{3b^3}{a^2} = 3a^{-2}b^3$.

5.(Source: BootCamp.13l, 14b, 15h)
a. Factor out the common factor and use the difference of cubes: $x^2y^2(y^3 - 8x^3) = x^2y^2(y^3 - (2x)^3) = x^2y^2(y - 2x)(y^2 + y(2x) + (2x)^2) = x^2y^2(y - 2x)(y^2 + 2xy + 4x^2)$.
b. $2x^2 + x - 28 = (2x - 7)(x + 4)$.
c. Group: $(2x^3 - 3x^2) + (-18x + 27) = x^2(2x - 3) - 9(2x - 3) = (x^2 - 9)(2x - 3) = (x - 3)(x + 3)(2x - 3)$.

6.(Source: BootCamp.19egk, 20cf) Tip: when multiplying fractions as in 1c, 6bc, resist the temptation to multiply out individual factors. Doing so only makes it more difficult to find and cancel any common factors.
a. Get a common denominator and add: $\frac{x+2}{x+2} \cdot \frac{1}{x-2} + \frac{1}{x+2} \cdot \frac{x-2}{x-2} = \frac{x+2+x-2}{(x-2)(x+2)} = \frac{2x}{(x-2)(x+2)}$, or, if you prefer, $\frac{2x}{x^2-4}$.
b. To multiply fractions, we don't need a common denominator. $\frac{x-3}{2x+1} \cdot \frac{x+3}{(x-1)(x-3)} = \frac{x+3}{(2x+1)(x-1)}$
c. To divide fractions, invert the divisor and multiply. $\frac{x^2+4}{(x-3)(x+3)} \cdot \frac{x+3}{x+2} = \frac{x^2+4}{(x-3)(x+2)}$.
($x^2 + 4$ doesn't factor over the reals. In particular, it is *not* $(x + 2)^2 = x^2 + 4x + 4$.)
d. $\frac{2}{(3x+1)(x-8)} - \frac{1}{(3x+1)(x+4)} = \frac{(x+4)}{(x+4)} \cdot \frac{2}{(3x+1)(x-8)} - \frac{1}{(3x+1)(x+4)} \cdot \frac{(x-8)}{(x-8)} = \frac{2(x+4)-(x-8)}{(x+4)(3x+1)(x-8)} = \frac{x+16}{(x+4)(3x+1)(x-8)}$.