1 (10 pts). Find all intercepts and asymptotes of \( y = \frac{2x^2 + 5x - 12}{2x - 1} \).
Find the \( x \)-coordinates of any points at which this curve intersects any of its asymptotes. You are not required to graph this curve.

**Solution:**

1. (Source: 3.5.more.1p) \( y \)-intercept: \( y = \frac{-12}{1} = 12 \) when \( x = 0 \).
   \( x \)-intercepts: \( y = \frac{2x^2 + 5x - 12}{2x - 1} = 0 \) implies \( 0 = 2x^2 + 5x - 12 = (2x - 3)(x + 4) \), or \( x = \frac{3}{2}, x = -4 \).
   The numerator and denominator have no factors in common, so a vertical asymptote occurs when the denominator is zero. That is, \( x = \frac{1}{2} \).
   The degree of numerator is one more than that of the denominator, so there’s a slant asymptote. To find it, perform long division.

\[
\begin{array}{c|cc}
& 2x & - 1 \\
\hline
2x & 2x^2 & + 5x \\
-2x^2 & - x & \hline
& 6x & - 12 \\
-6x & - 3 & \hline
& 0 & - 9
\end{array}
\]

(This means that \( \frac{2x^2 + 5x - 12}{2x - 1} = x + 3 - \frac{9}{2x - 1} \), which is \( \approx x + 3 \) when \( x \) is very large.)
Therefore, \( y = x + 3 \) is a slant asymptote.

There is no horizontal asymptote, since a rational function can’t have both an HA and an SA.

No rational function has a graph which intersects a vertical asymptote, since VAs can occur only at \( x \)-values at which the function is undefined. To see if the curve intersects its slant asymptote, solve:

\[
\frac{2x^2 + 5x - 12}{2x - 1} = x + 3
\]

\[
2x^2 + 5x - 12 = (x + 3)(2x - 1) = 2x^2 + 5x - 3
\]

\[ -12 = -3 \]

Finding no solutions, we conclude that the curve doesn’t intersect its SA. \( \text{(done)} \)

Note, we could also look for the intersection point by solving

\[
x + 3 - \frac{9}{2x - 1} = x + 3 \implies -\frac{9}{2x - 1} = 0,
\]

which again has no solutions.

For a nice picture of the graph, go to https://www.desmos.com/calculator/2zkqezm5ty