MATH 111–07 (Kunkle), Exam 0 Name: _____

50 pts, 50 minutes Sept 4, 2017

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(4 pts). Express as a fraction in lowest terms, if it exists.

a. $\frac{3}{10} + \frac{2}{15}$ b. $\frac{8}{5} \div 5$ c. $\frac{99}{25} \times \frac{5}{3}$ d. $\frac{2}{5} \div \frac{0}{2}$

2(2 pts). Rationalize the denominator and simplify the result: $\frac{3}{2+\sqrt{7}}$

3(6 pts). Rewrite the expression in simplest radical form or state that it does not exist.

a. $\sqrt{108}$ b. $\sqrt[3]{-378}$ c. $24^{3/2}$

4(9 pts). Rewrite the expression without parentheses. Write your answer to part a. without negative exponents. Write your answer to part b. without fractions (except possibly in an exponant).

a. $2x^3(3y^2)^3x^{-10}3^{-1}$ b. $\dfrac{\left(\frac{r}{s}\right)\left(\frac{s^2}{r^3}\right)}{\frac{3}{s} - \frac{7}{s}}$ c. $(uv^{-2}u^{-3}v^8)^{3/2}$

5(10 pts). Factor the polynomial completely over the real numbers.

a. $8x^3 + 27y^3$ b. $3x^5 - 24x^2$ c. $3x^3 - 8x^2 - 12x + 32$

6(4 pts). Simplify the rational function: $\dfrac{12x^2 - 11x - 15}{3x^2 + 4x - 15}$

7(15 pts). Perform the operation indicated and simplify the resulting rational function.

a. $\dfrac{1}{x-4} - \dfrac{3}{x-3}$

b. $\dfrac{x+2}{x^2+4} \cdot \dfrac{x-2}{x^2+4x+4}$

c. $\dfrac{1}{2x^2 - 11x + 15} + \dfrac{1}{2x^2 - 9x + 10}$

d. $\dfrac{\frac{2x+3}{3x-4} + 5}{x-1}$

1a.(Source: BootCamp.3d, 5m, 6p, 6.27) $\frac{3}{10}+\frac{2}{15}=\frac{3}{2\cdot5}+\frac{2}{3\cdot5}=\frac{3}{2\cdot5}\cdot\frac{3}{3}+\frac{2}{3\cdot5}\cdot\frac{2}{2}=\frac{9+4}{2\cdot3\cdot5}=\frac{13}{30}$.
1b. $\frac{8}{5}\div5=\frac{8}{5}\cdot\frac{1}{5}=\frac{8}{25}$.
1c. $\frac{99}{25}\times\frac{5}{3}=\frac{3\cdot33\cdot5}{5\cdot5\cdot3}=\frac{33}{5}$.
1d. $\frac{0}{2}=0$, so $\frac{2}{5}\div\frac{0}{2}$ does not exist.

2(2 pts).(Source: BootCamp.8b) Multiply top and bottom by the conjugate.
$\frac{3}{2+\sqrt{7}}\cdot\frac{2-\sqrt{7}}{2-\sqrt{7}}=\frac{3(2-\sqrt{7})}{2^2-\sqrt{7}^2}=\frac{3(2-\sqrt{7})}{4-7}=\frac{3(2-\sqrt{7})}{-3}=-(2-\sqrt{7})$, or $-2+\sqrt{7}$.

3(6 pts).(Source: BootCamp.9c, 10m, 11d, 11y) It helps to factor the radicand (the number under the radical) into prime numbers. This might take a few steps, as in part b.
3a. $\sqrt{108}=\sqrt{2^23^3}=\sqrt{2^23^23}=2\cdot3\sqrt{3}=6\sqrt{3}$.
3b. $\sqrt[3]{-378}=\sqrt[3]{-3\cdot126}=\sqrt[3]{-3\cdot3\cdot42}=\sqrt[3]{-3^2\cdot6\cdot7}=\sqrt[3]{-3^3\cdot2\cdot7}=-3\sqrt[3]{14}$.
3c. $24^{3/2}=(2^3\cdot3)^{3/2}=2^{9/2}\cdot3^{3/2}=2^4\cdot3^1\cdot2^{1/2}\cdot3^{1/2}=48\cdot6^{1/2}$, or $48\sqrt{6}$.

4a.(Source: BootCamp.12eir) $2x^3(3y^2)^3x^{-10}3^{-1}=2x^33^3(y^2)^3x^{-10}3^{-1}=2x^33^3y^6x^{-10}3^{-1}=2x^{-7}3^2y^6$. Without negative exponents, this is $\frac{18y^6}{x^7}$.
4b. $\left(\frac{r}{s}\cdot\frac{s^2}{r^3}\right)\div\frac{-4}{s}=\frac{s^1}{r^2}\div\frac{-4}{s}=\frac{s^1}{r^2}\cdot\frac{s}{-4}=\frac{s^2}{-4r^2}$. With negative exponents, we can rewrite this as $-4^{-1}r^{-2}s^2$.
4c. $(uv^{-2}u^{-3}v^8)^{3/2}=(u^{1-3}v^{-2+8})^{3/2}(u^{-2}v^6)^{3/2}=u^{-2\cdot3/2}v^{6\cdot3/2}=u^{-3}v^9$, or $\frac{v^9}{u^3}$.

5a.(Source: BootCamp.13km, 15g)
$8x^3+27y^3=(2x)^3+(3y)^3=(2x+3y)((2x)^2-(2x)(3y)+(3y)^2)=(2x+3y)(4x^2-6xy+9y^2)$.
The quadratic can't be factored further.
5b. Common factor. $3x^5-24x^2=3x^2(x^3-8)=3x^2(x^3-2^3)=3x^2(x-2)(x^2+2x+4)$.
The quadratic can't be factored further.
5c. Grouping. $3x^3-8x^2-12x+32=x^2(3x-8)-4(3x-8)=(x^2-4)(3x-8)=(x-2)(x+2)(3x-8)$.

6.(Source: BootCamp.18d) Factor the numerator and denominator, and cancel any common factors. $\frac{12x^2-11x-15}{3x^2+4x-15}=\frac{(3x-5)(4x+3)}{(3x-5)(x+3)}=\frac{4x+3}{x+3}$.

7a.(Source: BootCamp.19cj, 20cm) $\frac{(x-3)}{(x-3)}\frac{1}{(x-4)}-\frac{3}{(x-3)}\frac{(x-4)}{(x-4)}=\frac{(x-3)-3(x-4)}{(x-3)(x-4)}\overset{\star}{=}\frac{-2x+9}{(x-3)(x-4)}$.
Avoid the common mistake of cancelling $(x-3)$ or $(x-4)$ at $\star$. Neither of these is a factor of the top.
7b. $\frac{x+2}{x^2+4}\cdot\frac{x-2}{x^2+4x+4}=\frac{x+2}{x^2+4}\cdot\frac{x-2}{(x+2)^2}=\frac{x-2}{(x^2+4)(x+2)}$. Note that $x^2+4\neq(x+2)^2$.
7c. $\frac{1}{2x^2-11x+15}+\frac{1}{2x^2-9x+10}=\frac{1}{(x-3)(2x-5)}+\frac{1}{(2x-5)(x-2)}$
$=\frac{(x-2)}{(x-3)(2x-5)(x-2)}+\frac{(x-3)}{(x-3)(2x-5)(x-2)}=\frac{2x-5}{(x-3)(2x-5)(x-2)}=\frac{1}{(x-3)(x-2)}$.
7d. $\frac{\frac{2x+3}{3x-4}+5}{x-1}=\left(\frac{2x+3}{3x-4}+\frac{5(3x-4)}{3x-4}\right)\div(x-1)=\frac{2x+3+5(3x-4)}{3x-4}\cdot\frac{1}{(x-1)}=\frac{17x-17}{(3x-4)(x-1)}=\frac{17(x-1)}{(3x-4)(x-1)}=\frac{17}{3x-4}$.