1. Suppose that $\tan x = 12/5$ and that $\pi < x < 3\pi/2$. Find the following.
\[ \begin{align*}
&\text{a. } \sin x \\
&\text{b. } \cos x \\
&\text{c. } \cos(2x) \\
&\text{d. } \sin(2x) \\
&\text{e. } \cos \left( \frac{x}{2} \right)
\end{align*} \]

1. (Source: 4.5.31,47) $\pi < x < 3\pi/2$ means that the terminal side of $x$ is in Quadrant III. Choose vertical and horizontal legs so that both are negative and vertical over horizontal is 12/5. Then find the hypotenuse by the Pythagorean theorem.

\[ r^2 = 12^2 + 5^2 = 169 \Rightarrow r = 13. \]

The result:

\[
\begin{array}{c}
\text{13} \\
\text{12} \\
\text{5}
\end{array}
\]

a. $\sin x = \frac{-12}{13}$.

b. $\cos x = \frac{-5}{13}$.

c. $\cos(2x) = \cos^2 x - \sin^2 x = \left( \frac{-5}{13} \right)^2 - \left( \frac{-12}{13} \right)^2 = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}$.

d. $\sin(2x) = 2 \sin x \cos x = 2 \left( \frac{-12}{13} \right) \left( \frac{-5}{13} \right) = \frac{120}{169}$.

e. From the half-angle identity,

\[ \cos^2 \left( \frac{x}{2} \right) = \frac{1}{2} (1 + \cos x) = \frac{1}{2} \left( 1 + \frac{-5}{13} \right) = \frac{4}{13}. \]

\[ \cos \left( \frac{x}{2} \right) = \pm \frac{2}{\sqrt{13}}. \]

Divide all three sides of $\pi < x < 3\pi/2$ by 2 to obtain $\pi/2 < x/2 < 3\pi/4$. We conclude that $x/2$ is in Quadrant II and its cosine should be negative:

\[ \cos \left( \frac{x}{2} \right) = -\frac{2}{\sqrt{13}}. \]