MATH 111–15 (Kunkle), Quiz 9 Name: ______________________
10 pts, Take-home Due in class April 12, 2017

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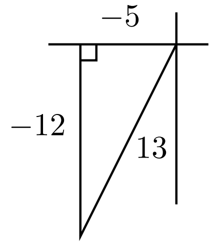
1 (10 pts). Suppose that $\tan x = 12/5$ and that $\pi < x < 3\pi/2$. Find the following.

a. $\sin x$ b. $\cos x$ c. $\cos(2x)$ d. $\sin(2x)$ e. $\cos\left(\frac{x}{2}\right)$

1.(Source: 4.5.31,47) $\pi < x < 3\pi/2$ means that the terminal side of x is in Quadrant III. Choose vertical and horizontal legs so that both are negative and vertical over horizontal is 12/5. Then find the hypotenuse by the Pythagorean theorem.

$$r^2 = 12^2 + 5^2 = 169 \Rightarrow r = 13.$$

The result:



a. $\sin x = \dfrac{-12}{13}$.

b. $\cos x = \dfrac{-5}{13}$.

c. $\cos(2x) = \cos^2 x - \sin^2 x = \left(\dfrac{-5}{13}\right)^2 - \left(\dfrac{-12}{13}\right)^2 = \dfrac{25}{169} - \dfrac{144}{169} = -\dfrac{119}{169}$.

d. $\sin(2x) = 2\sin x \cos x = 2\left(\dfrac{-12}{13}\right)\left(\dfrac{-5}{13}\right) = \dfrac{120}{169}$.

e. From the half-angle indentity,

$$\cos^2\left(\frac{x}{2}\right) = \frac{1}{2}(1+\cos x) = \frac{1}{2}\left(1 + \frac{-5}{13}\right) = \frac{4}{13}.$$
$$\cos\left(\frac{x}{2}\right) = \pm\frac{2}{\sqrt{13}}.$$

Divide all three sides of $\pi < x < 3\pi/2$ by 2 to obtain $\pi/2 < x/2 < 3\pi/4$. We conclude that $x/2$ is in Quadrant II and its cosine should be negative:

$$\cos\left(\frac{x}{2}\right) = -\frac{2}{\sqrt{13}}.$$