
1 (10 pts). Find all intercepts and asymptotes of $y = \frac{2x^2 + 5x - 12}{2x - 1}$.

You are not required to graph this curve.

Solution:

1. (Source: 3.5.more.1p) y -intercept: $y = \frac{-12}{-1} = 12$ when $x = 0$.

x -intercepts: $y = \frac{2x^2+5x-12}{2x-1} = 0$ implies $0 = 2x^2 + 5x - 12 = (2x - 3)(x + 4)$, or $x = 3/2, x = -4$.

The numerator and denominator have no factors in common, so a vertical asymptote occurs when the denominator is zero. That is, $x = 1/2$.

The degree of numerator is one more than that of the denominator, so there's a slant asymptote. To find it, perform long division.

$$\begin{array}{r} x + 3 \\ 2x - 1 \overline{) 2x^2 + 5x - 12} \\ \underline{-(2x^2 - x)} \\ 6x - 12 \\ \underline{-(6x - 3)} \\ -9 \end{array}$$

(This means that $\frac{2x^2+5x-12}{2x-1} = x + 3 - \frac{9}{2x-1}$, which is $\approx x + 3$ when x is very large.) Therefore, $y = x + 3$ is a slant asymptote.

(There is no horizontal asymptote, since a rational function can't have both an HA and an SA.)