1 (10 pts). Let $g(x) = \sqrt{5x - 1}$ and find the difference quotient

$$\frac{g(x + h) - g(x)}{h}.$$

Cancel the factor h from top and bottom.

Solution:

1. (Source: 2.10.more.1t)

$$\frac{g(x + h) - g(x)}{h}$$

$$= \frac{\sqrt{5(x + h) - 1} - \sqrt{5x - 1}}{h}$$

$$= \left(\frac{\sqrt{5x + 5h - 1} - \sqrt{5x - 1}}{h}\right)\left(\frac{\sqrt{5x + 5h - 1} + \sqrt{5x - 1}}{\sqrt{5x + 5h - 1} + \sqrt{5x - 1}}\right)$$

$$= \frac{(\sqrt{5x + 5h - 1})^2 - (\sqrt{5x - 1})^2}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}$$

$$= \frac{5x + 5h - 1 - (5x - 1)}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}$$

$$= \frac{5x + 5h - 1 - 5x + 1}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}$$

$$= \frac{5h}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}$$

$$= \frac{5}{\sqrt{5x + 5h - 1} + \sqrt{5x - 1}}$$

(done)