MATH 111–15 (Kunkle), Quiz 4 | Name: ______________________
10 pts, 10 minutes | Feb 14, 2017

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1 (10 pts). Find the two functions defined implicitly by the equation

$$y^2 + xy = -16.$$

Simplify your functions and give the domain of each.

1.(Source: 2.7.23)

Applying the quadratic formula to the equation

$$y^2 + xy + 16 = 0$$

with $a = 1$, $b = x$ and $c = 16$, we arrive at the solutions

$$y = \frac{-x \pm \sqrt{x^2 - 4 \cdot 16}}{2} = \frac{-x \pm \sqrt{x^2 - 64}}{2}$$

The two functions are

$$y = f(x) = \frac{1}{2}\left(-x + \sqrt{x^2 - 64}\right)$$

and

$$y = g(x) = \frac{1}{2}\left(-x - \sqrt{x^2 - 64}\right).$$

These have the same domain, since both require that $0 \leq x^2 - 64 = (x+8)(x-8)$. Sign chart:

$$\begin{array}{rl} x+8: & ---------0++++++++++++++++++ \\ x-8: & ---------------------0+++++++++ \\ x^2-64: & +++++++++0-----------0+++++++ \\ \hline x: & \qquad\qquad -8 \qquad\qquad\qquad 8 \end{array}$$

The domain of f and of g is the solution set to $x^2 - 64 \geq 0$, or$(-\infty, -8] \cup [8, \infty)$.

(done)

Comments:

1. The goal in this problem—and of every problem in section 2.7—is to represent the graph of the given xy-equation as the graphs of several functions $y = f(x)$. This requires **solving** for y, i.e., finding an equivalent equation in which y appears alone on one side and nowhere on the other side.
2. The most reliable way to solve a nonlinear inequality in general is with a sign chart. Ignore this advice at your own risk.
3. Finally, remember

$\sqrt{a \pm b}$ does **not** equal $\sqrt{a} \pm \sqrt{b}$.