No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

Unless a problem specifically refers to degrees, use radians in all questions and answers involving angles or trig functions. Evaluate any logs which can be expressed as a rational number (e.g., 2, $-5/4$, ...).

1(14 pts). Upsidaisium is a radioactive isotope. Suppose that a 20 g sample of Upsidaisium decays exponentially to 16 g in 30 days.

a. What will be the mass of the sample after 45 days?

b. What is the half-life of Upsidaisium?

2(14 pts). Find the values of $\sin t$ and $\cos t$ at the given values of t. Supporting work not required.

t	$\frac{5\pi}{4}$	$-\frac{\pi}{3}$	$\frac{13\pi}{6}$	$-\frac{19\pi}{6}$	9π
$\cos t$					
$\sin t$					

3(6 pts). Find the area of a circular sector of radius 2 cm and central angle 2.4 radians.

4(12 pts). Sketch the graph of $s(x) = \frac{2x^2+3x-2}{x^2+x-2}$.

Find and report all intercepts, asymptotes, and holes, or state that they do not exist. Give asymptotes by their equations; give holes and intercepts by their coordinates.

5(26 pts). Solve for x. Evaluate any logarithms in your answer, if possible. Express your answer to part a. in interval form.

a. $3^{x+2} \leq \dfrac{1}{9}$ b. $3^x = 4^{2x-1}$ c. $\log_6(x+8) + \log_6(x+3) = 1$

6(14 pts). Sketch the graph of the given function. Find and report all intercepts and asymptotes. Give asymptotes by their equations; give intercepts by their coordinates. State the domain and range of each function in interval form. Label your answers so I can tell which is which.

a. $u(x) = -e^x$ b. $w(x) = \log_2(x+8)$

7(10 pts). Evaluate the given expression.

a. $\log_{10}(10{,}000{,}000)$ b. $\log_{16}\left(\frac{1}{4}\right)$ c. $\ln\sqrt{e^3}$ d. $5^{\log_5 10}$

8(7 pts). Rewrite the expression as one logarithm: $\log_2\left(\dfrac{y}{x^2}\right) - 3\log_2(y^2) + 4\log_2 x$.

1.(Source: 5.4.16) Let $y = y_0 e^{kt}$ be the amount of Upsidaisium after t days. Take $t = 0$ to be the time when $y = 20$, so that $y_0 = 20$. Plug in $y = 16$ and $t = 30$ to solve for k:

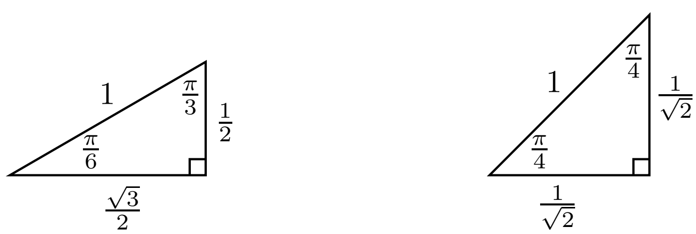
$$20e^{30k} = 16 \Rightarrow e^{30k} = 16/20 = 0.8 \Rightarrow 30k = \ln(0.8) \Rightarrow k = \frac{1}{30}\ln(0.8).$$

In part a., we're asked to calculate y when $t = 45$. That's $y = 20e^{\frac{45}{30}\ln(0.8)}$ (which, incidentally, is $20(0.8)^{3/2}$).

In part b., we set $y = 10$ and solve for t:

$$20e^{\frac{t}{30}\ln(0.8)} = 10 \Rightarrow e^{\frac{t}{30}\ln(0.8)} = 0.5 \Rightarrow \frac{t}{30}\ln(0.8) = \ln(0.5) \Rightarrow t = \frac{30\ln(0.5)}{\ln(0.8)}.$$

2.(Source: 4.2.11-32, 4.2.more, 4.4.1-13) Figure out the reference number of each angle and the quadrant containing its terminal point. Draw a vertical line from the terminal point to the x-axis and a line from the terminal point to the origin. Look for these two triangles.



t	$\frac{5\pi}{4}$	$-\frac{\pi}{3}$	$\frac{13\pi}{6}$	$-\frac{19\pi}{6}$	9π
quadrant	3	4	1	2	negative x-axis
reference number	$\pi/4$	$\pi/3$	$\pi/6$	$\pi/6$	0
$\cos t$	$-1/\sqrt{2}$	$1/2$	$\sqrt{3}/2$	$-\sqrt{3}/2$	-1
$\sin t$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	$1/2$	$1/2$	0

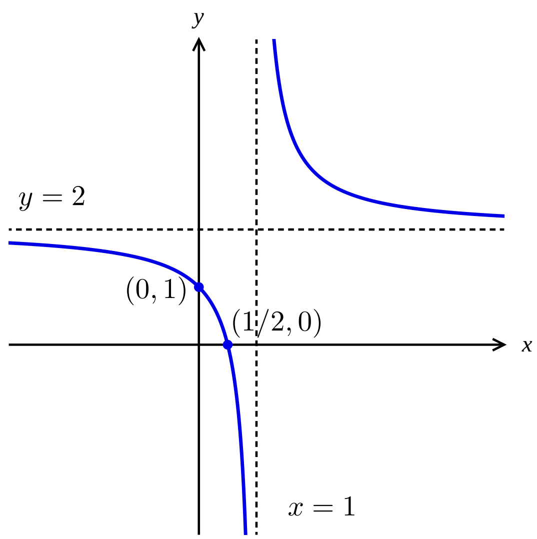
3.(Source: 4.1.72) $A = \frac{1}{2}r^2\theta = \frac{1}{2}2^2 \cdot 2.4 = 2 \cdot 2.4 = 4.8$ cm^2.

4.(Source: 3.5.more.1v) Factor: $s(x) = \frac{2x^2+3x-2}{x^2+x-2} = \frac{(2x-1)(x+2)}{(x-1)(x+2)}$. The common factor $x + 2$ means the graph of $s(x)$ is the same as that of $\frac{2x-1}{x-1}$, except that $y = s(x)$ is undefined at $x = -2$. Now graph $y = \frac{2x-1}{x-1}$

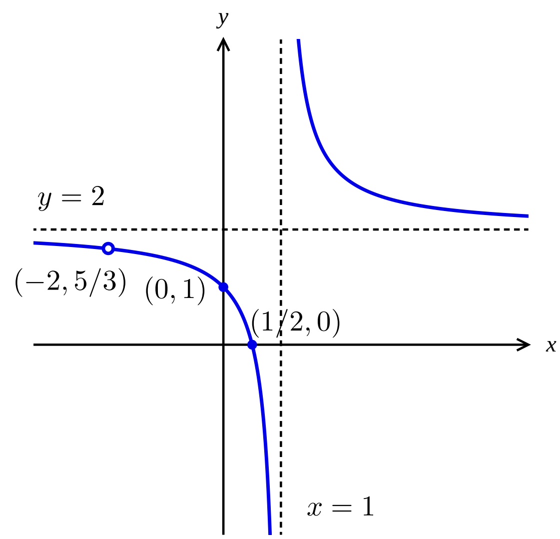
There's a vertical asymptote at $x = 1$. Since the degree of the numerator and of the denominator are the same, there is no slant asymptote, and the horizontal asymptote is $y = 2$ (= lead coefficient on top/lead coefficient on bottom). At $x = 0$ we calculate $y = 1$, and solving $y = 0$ we obtain $2x - 1 = 0 \Rightarrow x = 1/2$, so the y-intercept is $(0, 1)$, and the only x-intercept is $(1/2, 0)$.

To graph, it helps to make a sign chart:

$2x-1:$	$-----------$	0	$++++++++++++++++$		$++++++++$
$x-1:$	$-----------$		$-----------$	0	$++++++++$
$\dfrac{2x-1}{x-1}:$	$++++++++++$	0	$-----------$	VA	$++++++++$
$x:$		$\frac{1}{2}$		1	



$$y = \frac{2x-1}{x-1}$$



$$y = \frac{(2x-1)(x+2)}{(x-1)(x+2)}$$

Combining all the information we have generates a graph for $y = \frac{2x-1}{x-1}$. Punch out a hole from this graph at $x = -2$ to obtain the graph of $y = \frac{(2x-1)(x+2)}{(x-1)(x+2)}$. The y-coordinate of the hole is the altitude of $y = \frac{2x-1}{x-1}$ at $x = -2$, or $5/3$.

5a.(Source: 5.1.more.2b) $3^{x+2} \leq \frac{1}{9} = 3^{-2}$. Note that 3^x is an increasing function, so larger output implies larger input: $x + 2 \leq -2$, or $x \leq -4$. The solution set is $(-\infty, -4]$.

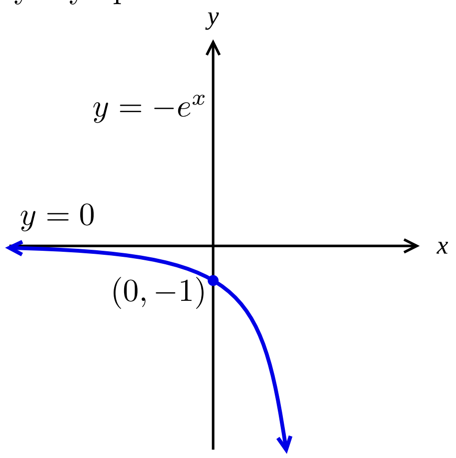
5b.(Source: 5.3.19) To get the variable our of the exponent, take the natural log of both sides: $\ln 3^x = \ln 4^{2x-1} \Rightarrow x \ln 3 = (2x-1)\ln 4$. Distribute and get the x's to one side:

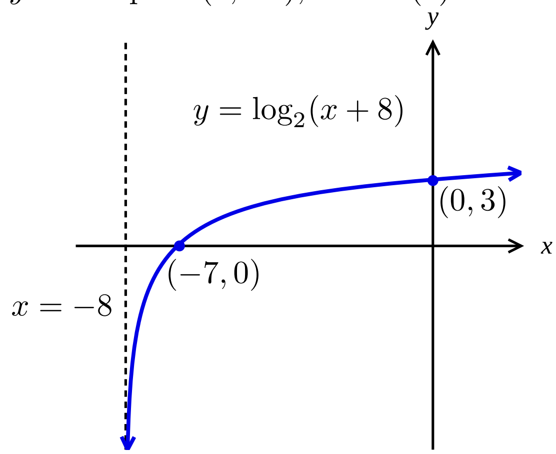
$$\begin{aligned} x\ln 3 &= x2\ln 4 - \ln 4 \\ \ln 4 &= x2\ln 4 - x\ln 3 = x(2\ln 4 - \ln 3) \end{aligned}$$

Now divide to solve: $x = (\ln 4)/(2\ln 4 - \ln 3)$.

5c.(Source: 5.3.more.3c) Combine the logs: $\log_6(x+8)(x+3) = 1$ and write in exponential form: $(x+8)(x+3) = 6^1 = 6$. To solve the quadratic, get 0 on one side and factor the other: $0 = x^2 + 11x + 24 - 6 = x^2 + 11x + 18 = (x+2)(x+9)$, which gives $x = -2$ and $x = -9$. Of these, -9 is not in the domain of the functions in the original equation, since $\log_6(-1)$ is not defined. The only solution is $x = -2$.

6a.(Source: 5.2.7,19. 5.3.more.3i) To graph $u(x) = -e^x$, reflect the graph of $y = e^x$ across the x-axis. Domain of $u(x)$ is $(-\infty, \infty)$ and the range is $(-\infty, 0)$. The x-axis, or $y = 0$, is the only asymptote. There's no x-intercept; the y-intercept is $(0, -1)$, since $u(0) = -e^0 = -1$.





6b.(Source: 5.2.26) To graph $w(x) = \log_2(x + 8)$, shift the graph of $y = \log_2 x$ to the left 8 units. Domain of w is $(-8, \infty)$. Range of w is $(-\infty, \infty)$. The vertical line $x = -8$ is the only asymptote. The x-intercept is $(-7, 0)$, since $\log_2 1 = 0$. To find the y-intercept, we calculate $w(0) = \log_2 8$. This equals 3, since $8 = 2^3$.

7.(Source: 5.2.13,16,18,21) $\log_b x = y$ means $x = b^y$. Consequently, $b^{\log_b x} = x$ for all $x > 0$, and $\log_b b^y = y$ for all y.

a. $\log_{10}(10{,}000{,}000) = 7$ because $10{,}000{,}000 = 10^7$.

b. $\log_{16}\left(\frac{1}{4}\right) = -1/2$ because $\frac{1}{4} = \frac{1}{\sqrt{16}} = \frac{1}{16^{1/2}} = 16^{-1/2}$.

c. $\ln\sqrt{e^3} = \ln e^{3/2} = 3/2$.

d. $5^{\log_5 10} = 10$.

8.(Source: 5.2.54) Combine using the properties of logs.

$$\log_2\left(\frac{y}{x^2}\right) - 3\log_2(y^2) + 4\log_2 x = \log_2\left(\frac{y}{x^2}\right) - \log_2((y^2)^3) + \log_2 x^4$$

$$= \log_2\left(\frac{y}{x^2}\right) + \log_2\left(\frac{x^4}{y^6}\right) = \log_2\left(\frac{y}{x^2}\frac{x^4}{y^6}\right) = \log_2\left(\frac{x^2}{y^5}\right)$$