MATH 111–03 (Kunkle), Exam 1 Name: ______________________________

100 pts, 75 minutes Feb 7, 2017

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(17 pts). Solve for x. Write the solution set in interval form.

a. $2x^2 + 23x < 12$ b. $|3x+2| \geq 5$ c. $|3x+2| \leq 5$

2(5 pts). Find an equation of the circle centered at $(3, -2)$ passing through the point $(1, 4)$.

3(2 pts). Find an equation of the line passing through the point $(3, 4)$ parallel to the x-axis.

4(19 pts). Let $p(x) = 4x^2 - 16x + 13$.

a. Write $p(x)$ in standard form.
b. Find the coordinates of the vertex and all intercepts of the graph of $p(x)$.
c. Find the equation of the axis of symmetry of the graph of $p(x)$.
d. Find the minimum and maximum values of $p(x)$, if either exist.

5(24 pts). Simplify and cancel the common factor. (In c., first rationalize the numerator.)

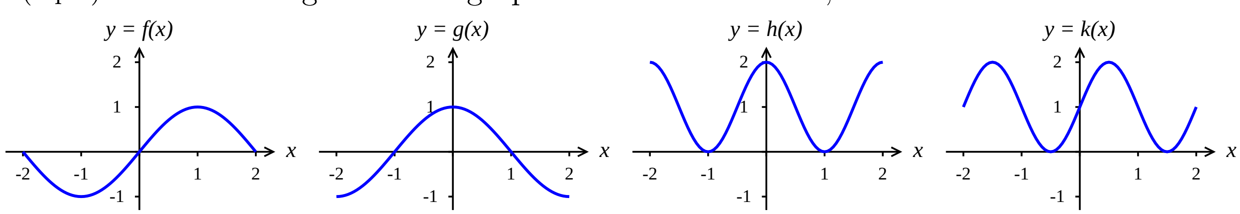
a. $\dfrac{1}{x+3} + \dfrac{3}{2x^2+9x+9}$ b. $\dfrac{(1-x)^3 - (1+x)^3}{x}$ c. $\dfrac{\sqrt{x}-4}{x-16}$

6(16 pts). Let $f(x) = \frac{x}{x+2}$ and $g(x) = \frac{x}{x-2}$. Find the following.

a. The domain of $f(x)$. b. $\big(f \div g\big)(x)$ and its domain.
c. $\big(f \circ g\big)(x)$ and its domain.

Work in the space below and label your answers. Simplify all rational functions, and state all domains in interval form.

7(4 pts). Label each figure as the graph of an even function, an odd function or neither.



8(2 pts). Sketch the graph of $y = |f(x)|$, where f is the function in Problem 7a.

9(11 pts). Graph the function and find all intercepts and asymptotes, if any. Your drawing needn't be to scale, but it should clearly show how b. is obtained from a., as well as the intercepts and asymptotes you've found.

a. $y = x^{3/2}$ b. $y = -(x+4)^{3/2} + 27$

1.(Source: 1.1.41, 1.2.more.1a, 2ab)
1a. (8 points) Get 0 on one side and factor the other: $0 > 2x^2+23x-12 = (2x-1)(x+12)$. Now make a sign chart:

$$\begin{array}{rl} x+12: & - - - - - - - 0 + + + + + + + + + + + + + + \\ 2x-1: & - - - - - - - - - - - - - - - 0 + + + + + + \\ (2x-1)(x+12): & + + + + + + + 0 - - - - - - - - 0 + + + + + + \\ \hline x: & \qquad\qquad -12 \qquad\qquad\qquad 1/2 \end{array}$$

The solution set is $(-12, 1/2)$.
1b&c. (9 points combined) These two parts go hand in hand, and there are the multiple correct solutions. Here's one. $|3x+2| \leq 5$ means that $3x+2$ is a number whose distance from 0 is less than 5, so that $3x+2$ lies between -5 and 5:

$$-5 < 3x+2 < 5 \Rightarrow -7 < 3x < 3 \Rightarrow -7/3 < x < 1$$

so the solution set of c. $|3x+2| \leq 5$ is $[-7/3, 1]$. This also shows that $|3x+2| = 5$ at $x = -7/3$ and $x = 1$. Therefore, the solution of b. $|3x+2| \geq 5$ must be $(-\infty, -7/3] \cup [1, \infty)$.

2.(Source: 1.4.22) The radius of the circle is the distance from its center to the point on its circumference: $r = \sqrt{(3-1)^2 + (4-(-2))^2} = \sqrt{40} \Rightarrow r^2 = 40$. The circle has the equation $(x-3)^2 + (y+2)^2 = 40$.

3.(Source: 2.3.31) Since it is parallel to the x-axis, the line in question is horizontal. Its equation is $y = 4$.

4.(Source: 2.4.more.1j) Let $p(x) = 4x^2 - 16x + 13$.
4a. (7 points) $p(x) = 4(x^2-4x)+13 = 4(x^2-4x+4)-16+13 = 4(x-2)^2-3$.
4b. (8 points) From its standard form, we see that the graph of p is obtained by shifting $y = x^2$ to the right 2 units, stretching vertically, and shifting down 3 units. That puts the vertex at $(2, -3)$.

To find the y-intercept, set $x = 0$ and calculate $y = 13$. To find the x-intercept, set $y = 0$ and solve for x. This is most easily done with the standard form we've already found:

$$4(x-2)^2-3 = 0 \Rightarrow 4(x-2)^2 = 3 \Rightarrow (x-2)^2 = 3/4 \Rightarrow x-2 = \pm\sqrt{3/4} \Rightarrow x = 2 \pm \sqrt{3}/2.$$

That is, the y-intercept is $(0, 13)$ and the x-intercepts are $(2+\sqrt{3}/2, 0)$ and $(2-\sqrt{3}/2, 0)$.
4c. (2 points) The axis of symmetry is the vertical line through the vertex, or $x = 2$.
4d. (2 points) Because the graph of p opens upward, the minimum value of $p(x)$ is -3, its value at the vertex. $p(x)$ has no maximum value.

5.(Source: 1.5.18, 21, 27)

5a. (8 points) Factoring, the second fraction equals $\frac{3}{(x+3)(2x+3)}$. Get a common denominator to add the fractions:

$$\frac{1}{x+3}+\frac{3}{(x+3)(2x+3)}=\frac{2x+3}{2x+3}\cdot\frac{1}{x+3}+\frac{3}{(x+3)(2x+3)}=\frac{2x+3+3}{(x+3)(2x+3)}$$
$$=\frac{2x+6}{(x+3)(2x+3)}=\frac{2(x+3)}{(x+3)(2x+3)}=\frac{2}{2x+3}.$$

5b. (11 points) It's easiest to expand $(1-x)^3$ and $(1+x)^3$ by using Pascal's triangle. Because its third row is 1 3 3 1,

$$(1+x)^3=1+3x+3x^2+x^3$$
$$(1-x)^3=1-3x+3x^2-x^3$$

and so

$$\frac{(1-x)^3-(1+x)^3}{x}=\frac{(1-3x+3x^2-x^3)-(1+3x+3x^2+x^3)}{x}$$
$$=\frac{1-3x+3x^2-x^3-1-3x-3x^2-x^3}{x}$$
$$=\frac{-6x-2x^3}{x}=\frac{x(-6-2x^2)}{x}=-6-2x^2$$

5c. (5 points) $\left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right)=\frac{\sqrt{x}^2-4^2}{(x-16)(\sqrt{x}+4)}=\frac{x-16}{(x-16)(\sqrt{x}+4)}=\frac{1}{\sqrt{x}+4}.$

6.(Source: 2.1.15, 2.6.6, 2.6.more2k)

6a. (2 points) We can divide by any number except zero, so the domain of $f(x)=\frac{x}{x+2}$ is all x except -2. In interval notation, that's $(-\infty,-2)\cup(-2,\infty)$.

6b. (6 points) $\big(f\div g\big)(x)=\frac{\frac{x}{x+2}}{\frac{x}{x-2}}$ is defined as long as none of the three denominators $x+2$, $x-2$, and $\frac{x}{x-2}$ is zero. Remember that for a fraction to equal zero its numerator must equal zero. So the domain is all x except 0 and ± 2. In interval notation, $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$.

Now that we've found its domain, it's safe to simplify: $\frac{\frac{x}{x+2}}{\frac{x}{x-2}}=\frac{x}{x+2}\cdot\frac{x-2}{x}=\frac{x-2}{x+2}$

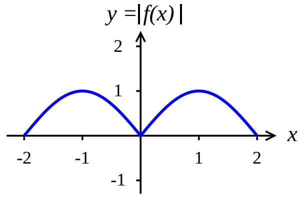
6c. (8 points) For $\big(f\circ g\big)(x)=\frac{g(x)}{g(x)+2}=\frac{\frac{x}{x-2}}{\frac{x}{x-2}+2}$ to be defined, we need the two denominators $x-2$ and $\frac{x}{x-2}+2=\frac{x+2(x-2)}{x-2}=\frac{3x-4}{x-2}$ to be nonzero. The domain is all x except 2 and 4/3. In interval form, $(-\infty,4/3)\cup(4/3,2)\cup(2,\infty)$.

Now that we've found its domain, it's safe to simplify:

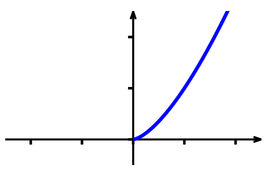
$$\frac{\frac{x}{x-2}}{\frac{x}{x-2}+2}=\frac{\frac{x}{x-2}}{\frac{3x-4}{x-2}}=\frac{x}{x-2}\times\frac{x-2}{3x-4}=\frac{x}{3x-4}$$

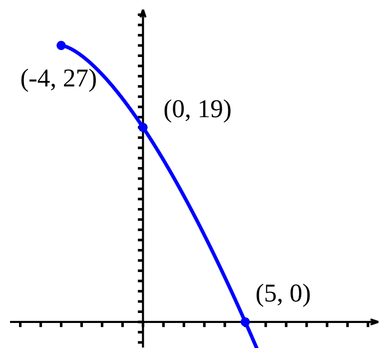
7.(Source: 2.2.12, 14) A function is even (odd) if its graph is symmetric across the y-axis (origin). a. Odd. b. Even. c. Even. d. Neither.

8.(Source: 2.5.25-30) The graph of $|f(x)|$ is obtained by reflecting the part of the graph of $f(x)$ that lies below the x-axis across that axis:



9.(Source: 2.2.more.1o) Start with the graph of $y = x^{3/2} = (\sqrt{x})^3$. This looks roughly like $y = x^2$ is quadrant 1. $\sqrt{x}$ is defined only for nonnegative x-values, so the entire graph is in quadrant 1.





a. (2 points) $y = x^{3/2}$ b. (9 points) $y = -(x+4)^{3/2} + 27$

To obtain the graph of $y = -(x+4)^{3/2} + 27$, reflect $y = x^{3/2}$ across the x-axis, and shift the result up 27 and to the left 4. See the graph above. To illustrate this shift, it's helpful to label $(-4, 27)$ as the new "vertex."

Set $x = 0$ and calculate $y = -4^{3/2} + 27 = -\sqrt{4}^3 + 27 = -8 + 27 = 19$. Therefore the y-intercept is $(0, 19)$.

Now set $y = 0$ solve for x.

$$\begin{aligned} 0 &= -(x+4)^{3/2} + 27 \\ (x+4)^{3/2} &=^* 27 \\ (x+4)^{1/2} &= 27^{1/3} = 3 \\ (x+4) &= 3^2 = 9 \\ x &= 9 - 4 = 5. \end{aligned}$$

so the x-intercept is $(5, 0)$.

Avoid large numbers. After the step marked *, I took the cube root and then squared. If you squared before taking the cube root, you were left having to calculate $\sqrt[3]{729}$.