

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(7 pts). Express as a fraction in lowest terms, if it exists.

a. $\frac{3}{20} + \frac{4}{3}$ b. $\frac{3}{12} + \frac{11}{28}$ c. $\frac{8}{3} \times \frac{9}{4}$ d. $\frac{9}{5} \div \frac{36}{125}$

2(5 pts). Rationalize the denominator and simplify the result.

a. $\frac{5}{2\sqrt{6}}$ b. $\frac{14}{6-\sqrt{12}}$

3(7 pts). Rewrite the expression in simplest radical form or state that it does not exist.

a. $\sqrt[3]{-320}$ b. $(-81)^{5/2}$ c. $125^{-2/3}$ d. $32^{3/4}$

4(9 pts). Rewrite the expression without parentheses. Write your answer to part a. without negative exponents. Write your answer to b. without fractions (except possibly in an exponent).

a. $x^9((2x^{-1}z^3)^2)^3y^{-2}(4y)^{-3}$ b. $\frac{w^2 \left(\frac{uw^{-1}}{u^{-1}w} \right)}{\left(\frac{u}{w^{1/2}} \right)^{-3}}$ c. $(3(x-y))^2$

5(10 pts). Factor the polynomial completely over the real numbers.

a. $27 - u^3$ b. $18x^2 - 39x + 15$ c. $x^5 - 2x^4 + 8x^2 - 16x$

6(5 pts). Simplify the rational function: $\frac{5x^3-2x^2+45x-18}{5x^2+13x-6}$

7(12 pts). Perform the operation indicated and simplify the resulting rational function.

a. $\frac{2}{x+3} - \frac{5}{3x^2+8x-3}$ b. $\frac{2}{x+5} \cdot \frac{x^2+x-20}{x-20}$ c. $\frac{\frac{x}{x-4} + \frac{1}{3}}{x^2-1}$

1.(Source: BootCamp.2,3,5,6)

a. $\frac{3}{20} + \frac{4}{3} = \frac{3}{3} \cdot \frac{3}{20} + \frac{4}{3} \cdot \frac{20}{20} = \frac{9}{60} + \frac{80}{60} = \frac{89}{60}$.

b. $\frac{3}{12} + \frac{11}{28} = \frac{1}{4} + \frac{11}{28} = \frac{7}{7} \cdot \frac{1}{4} + \frac{11}{28} = \frac{18}{28} = \frac{9 \cdot 2}{14 \cdot 2} = \frac{9}{14}$.

c. $\frac{8}{3} \times \frac{9}{4} = \frac{2 \cdot 4 \cdot 3 \cdot 3}{3 \cdot 4} = \frac{2 \cdot 3}{1} = 6$.

d. $\frac{9}{5} \div \frac{36}{125} = \frac{9}{5} \times \frac{125}{36} = \frac{9 \cdot 5 \cdot 25}{5 \cdot 9 \cdot 4} = \frac{25}{4}$. (Don't take the square root: $\frac{25}{4} \neq \frac{5}{2}$.)

2a.(Source: BootCamp.7e) $\frac{5}{2\sqrt{6}} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{2 \cdot 6} = \frac{5\sqrt{6}}{12}$.

2b.(Source: BootCamp.8j) Can simplify the fraction first: $\frac{14}{6-\sqrt{12}} = \frac{14}{6-\sqrt{4 \cdot 3}} = \frac{14}{6-\sqrt{4}\sqrt{3}} = \frac{14}{6-2\sqrt{3}} = \frac{14}{2(3-\sqrt{3})} = \frac{7}{3-\sqrt{3}}$. Now use the conjugate: $\frac{7}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{7(3+\sqrt{3})}{9-3} = \frac{7(3+\sqrt{3})}{6}$.

3a.(Source: BootCamp.10f) $\sqrt[3]{-320} = \sqrt[3]{-32 \cdot 10} = \sqrt[3]{-2^6 \cdot 5} = \sqrt[3]{-1} \sqrt[3]{2^6} \sqrt[3]{5} = -2^2 \sqrt[3]{5} = -4 \sqrt[3]{5}$.

3b.(Source: BootCamp.11f) $(-81)^{5/2} = (\sqrt{-81})^5$ does not exist.

3c.(Source: BootCamp.11g) $125^{-2/3} = \frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2} = \frac{1}{25}$.

3d.(Source: BootCamp.11.28) $32^{3/4} = (2^5)^{3/4} = 2^{15/4} = 2^{3+3/4} = 2^3 \cdot 2^{3/4} = 8\sqrt[4]{8}$.

4a.(Source: BootCamp.12h) Note that $((2x^{-1}z^3)^2)^3 = (2x^{-1}z^3)^6 = 2^6 x^{-6} z^{18}$ and $(4y)^{-3} = (2^2 y)^{-3} = 2^{-6} y^{-3}$. Now $x^9 ((2x^{-1}z^3)^2)^3 y^{-2} (4y)^{-3} = x^9 2^6 x^{-6} z^{18} y^{-2} 2^{-6} y^{-3}$. Add exponents: $= x^{9-6} 2^{6-6} z^{18} y^{-2-3} = x^3 z^{18} y^{-5} = \frac{x^3 z^{18}}{y^5}$.

4b.(Source: BootCamp.12r) First observe that $\left(\frac{uw^{-1}}{u^{-1}w}\right) = \frac{u^2}{w^2}$ and that $\left(\frac{u}{w^{1/2}}\right)^{-3} = \frac{u^{-3}}{w^{-3/2}} = \frac{w^{3/2}}{u^3}$. Now $\frac{w^2 \left(\frac{uw^{-1}}{u^{-1}w}\right)}{\left(\frac{u}{w^{1/2}}\right)^{-3}} = \frac{w^2}{1} \cdot \frac{u^2}{w^2} \div \frac{w^{3/2}}{u^3} = \frac{u^2}{1} \cdot \frac{u^3}{w^{3/2}} = \frac{u^5}{w^{3/2}} = u^5 w^{-3/2}$.

4c.(Source: BootCamp.12m)

$$(3(x-y))^2 = 3^2(x-y)^2 = 9(x^2 - 2xy + y^2) = 9x^2 - 18xy + 9y^2.$$

5.(Source: BootCamp.13k) $27 - u^3 = 3^3 - u^3 = (3-u)(3^2 + 3u + u^2) = (3-u)(9 + 3u + u^2)$. The quadratic doesn't factor further, so we're done.

5a.(Source: BootCamp.14cd) It helps to look for common factors first.

$$18x^2 - 39x + 15 = 3(6x^2 - 13x + 5) = 3(2x-1)(3x-5).$$

5b.(Source: BootCamp.15n) $x(x^4 - 2x^3 + 8x - 16) = x(x^3(x-2) + 8(x-2))$

$$= x(x-2)(x^3 + 8) = x(x-2)(x+2)(x^2 - 2x + 4).$$

6.(Source: BootCamp.16gj) $\frac{5x^3 - 2x^2 + 45x - 18}{5x^2 + 13x - 6} = \frac{x^2(5x-2) + 9(5x-2)}{(5x-2)(x+3)} = \frac{(x^2+9)(5x-2)}{(5x-2)(x+3)} = \frac{(x^2+9)}{(x+3)}$.

This fraction does not simplify further. In particular, $x^2 + 9 \neq (x+3)^2$.

7a.(Source: BootCamp.17i) $\frac{2}{x+3} - \frac{5}{3x^2+8x-3} = \frac{2}{x+3} - \frac{5}{(3x-1)(x+3)} = \frac{3x-1}{3x-1} \cdot \frac{2}{x+3} - \frac{5}{(3x-1)(x+3)} = \frac{2(3x-1)-5}{(3x-1)(x+3)} = \frac{6x-7}{(3x-1)(x+3)}$.

7b.(Source: BootCamp.18c) $\frac{2}{x+5} \cdot \frac{x^2+x-20}{x-20} = \frac{2}{x+5} \cdot \frac{(x+5)(x-4)}{x-20} = \frac{2(x-4)}{x-20}$.

7c.(Source: BootCamp.18k) $\frac{\frac{x-4}{x^2-1} + \frac{1}{3}}{\frac{x-4}{x^2-1}} = \left(\frac{x}{x-4} + \frac{1}{3}\right) \cdot \frac{1}{x^2-1} = \left(\frac{3}{3} \cdot \frac{x}{x-4} + \frac{1}{3} \cdot \frac{x-4}{x-4}\right) \cdot \frac{1}{(x-1)(x+1)} = \frac{4x-4}{3(x-4)} \cdot \frac{1}{(x-1)(x+1)} = \frac{4(x-1)}{3(x-4)(x-1)(x+1)} = \frac{4}{3(x-4)(x+1)}$.