1 (10 pts). Find all intercepts and asymptotes of \( y = \frac{2x^2 + 5x - 12}{2x - 1} \).

You are not required to graph this curve.

Solution:

1. (Source: 3.5.more.1p) \( y \)-intercept: \( y = \frac{-12}{-1} = 12 \) when \( x = 0 \).

   \( x \)-intercepts: \( y = \frac{2x^2 + 5x - 12}{2x - 1} = 0 \) implies \( 0 = 2x^2 + 5x - 12 = (2x - 3)(x + 4) \), or \( x = \frac{3}{2}, x = -4 \).

   The numerator and denominator have no factors in common, so a vertical asymptote occurs when the denominator is zero. That is, \( x = \frac{1}{2} \).

   The degree of numerator is one more than that of the denominator, so there’s a slant asymptote. To find it, perform long division.

   \[
   \begin{array}{r|ll}
   & x & + 3 \\
   \hline
   2x - 1 & 2x^2 & + 5x & - 12 \\
   & (2x^2 & - x) \\
   \cline{2-3}
   & 6x & - 12 \\
   & -(6x & - 3) \\
   \cline{2-3}
   & & 9 \\
   \end{array}
   \]

   (This means that \( \frac{2x^2 + 5x - 12}{2x - 1} = x + 3 - \frac{9}{2x - 1} \), which is \( \approx x + 3 \) when \( x \) is very large.) Therefore, \( y = x + 3 \) is a slant asymptote.

   (There is no horizontal asymptote, since a rational function can’t have both an HA and an SA.)