1a (9 pts). Find \( g'(x) \) if \( g(x) = \sqrt{5x - 1} \).

1b (1 pts). Use your answer to part a to find the slope of the line tangent to \( y = g(x) \) at \( x = 2 \).

Solution:

1a. (Source: 2.9.more.1t)

\[
g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h}
= \lim_{h \to 0} \frac{\sqrt{5(x + h) - 1} - \sqrt{5x - 1}}{h}
= \lim_{h \to 0} \left( \frac{\sqrt{5x + 5h - 1} - \sqrt{5x - 1}}{h} \right) \left( \frac{\sqrt{5x + 5h - 1} + \sqrt{5x - 1}}{\sqrt{5x + 5h - 1} + \sqrt{5x - 1}} \right)
= \lim_{h \to 0} \frac{(\sqrt{5x + 5h - 1})^2 - (\sqrt{5x - 1})^2}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}
= \lim_{h \to 0} \frac{5x + 5h - 1 - (5x - 1)}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}
= \lim_{h \to 0} \frac{5x + 5h - 1 - 5x + 1}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}
= \lim_{h \to 0} \frac{5h}{h(\sqrt{5x + 5h - 1} + \sqrt{5x - 1})}
= \lim_{h \to 0} \frac{5}{\sqrt{5x + 5h - 1} + \sqrt{5x - 1}}
= \frac{5}{2\sqrt{5x - 1}}.
\]

1b. The slope of the line tangent to \( y = g(x) \) at \( x = 2 \) is \( g'(2) = \frac{5}{2\sqrt{5 \cdot 2 - 1}} = \frac{5}{2\sqrt{9}} = \frac{5}{6} \).

Comment: \( g'(x) \) (Section 2.9) and \( g^{-1}(x) \) (Section 2.7) are not the same! Read these two sections of the text and review the homework you did from each. Make sure you recognize the difference between these symbols.