No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

A mistake early in your solution does not rule out your receiving full credit for later steps.

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1 (12 pts). Sketch one cycle of the graph of

$$y = \cos\left(3x - \frac{\pi}{4}\right).$$

Draw the axes where you wish. Use and label the hashmarks so that your graph clearly shows the amplitude and the locations of all maximums, minimums, and zeros of the function that occur on your cycle.

2 (12 pts). Evaluate:

a. $\tan(\frac{\pi}{3})$ b. $\cot(-\frac{3\pi}{2})$ c. $\sec(\frac{3\pi}{4})$ d. $\csc(-\frac{7\pi}{6})$

3 (21 pts). Suppose that $\sin x = \frac{1}{4}$ and $\frac{\pi}{2} \le x \le \pi$. Find the following:

a. $\cos x$ b. $\csc x$ c. $\cos(2x)$ d. $\sin(\frac{x}{2})$

4 (19 pts). Evaluate:

a. $\sin(\frac{7\pi}{12})$ b. $\tan^{-1}(-\frac{1}{\sqrt{3}})$ c. $\cos^{-1}(\cos(\frac{6\pi}{5}))$ d. $\tan^{-1}(\tan(\frac{6\pi}{5}))$

5 (14 pts). Find all solutions t to the equation: $2\cos^2 t + \sin t - 1 = 0$

6 (12 pts). A 20-ft ladder is leaning against a (very tall) wall. Assume that the wall is vertical and the ground is horizontal.

a. If the ladder makes a 70° angle **with the ground**, how high is the top of the ladder above the ground?

b. I moved the ladder, and now its base is 8 ft from the wall. What angle does the ladder now make **with the wall**?

7 (10 pts). Sketch one cycle of the graph of

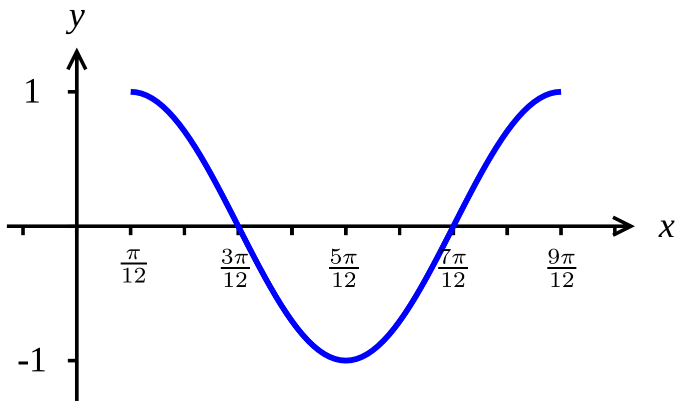
$$y = -2\csc x.$$

Draw the axes where you wish. Use and label the hashmarks so that your graph clearly shows the distinguishing characteristics (minimums, maximums, asymptotes) of this curve.

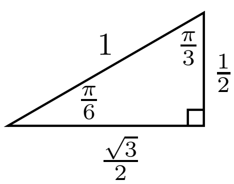
1. (Source: 4.3.34) The function $\cos\left(3x - \frac{\pi}{4}\right)$ will go through one cycle when the angle inside the cosine goes from 0 to 2π.

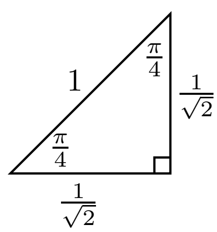
$$0 \le 3x - \frac{\pi}{4} \le 2\pi \qquad \frac{1}{3}\cdot\frac{\pi}{4} \le x \le \frac{1}{3}\cdot\frac{9\pi}{4}$$
$$\frac{\pi}{4} \le 3x \le 2\pi + \frac{\pi}{4} = \frac{9\pi}{4} \qquad \frac{\pi}{12} \le x \le \frac{9\pi}{12}$$

So, draw a cycle of the cosine starting at $x = \frac{\pi}{12}$ and ending at $x = \frac{9\pi}{12}$ $(= \frac{3\pi}{4})$.



2. (Source: 4.4.1-2) Figure out the reference number of each angle and the quadrant containing its terminal point. Draw a vertical line from the terminal point to the x-axis and a line from the terminal point to the origin. Look for these two triangles.





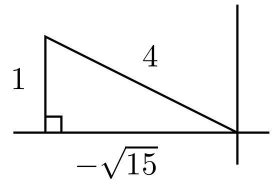
a. Quadrant I: $\tan(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3}$.

b. North pole: $\cot(-\frac{3\pi}{2}) = 0 \div 1 = 0$.

c. Quadrant II: $\sec(\frac{3\pi}{4}) = 1 \div \cos(\frac{3\pi}{4}) = -\sqrt{2}$.

d. Quadrant II: $\csc(-\frac{7\pi}{6}) = 1 \div \sin(-\frac{7\pi}{6}) = 2$

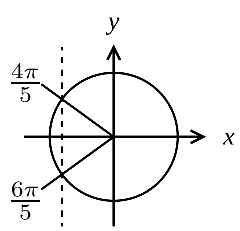
3. (Source: 4.4.23, 4.5.35, 4.5.49) $\frac{\pi}{2} \le x \le \pi$ tells us that the terminal side of x is in Quadrant II. Choose vertical leg and hypotenuse so vertical over hypotenuse is 1/4. Then find the horizontal leg by the Pythagorean theorem. Remember that the horizontal is negative in Quadrant II: $4^2 = 1^2 + x^2 \Rightarrow x = -\sqrt{15}$. The result:

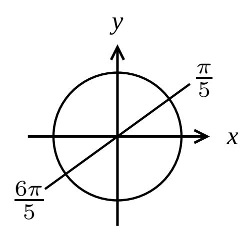


a. $\cos x = \frac{-\sqrt{15}}{4}$ b. $\csc x = (\sin x)^{-1} = 4$ c. $\cos(2x) = \cos^2 x - \sin^2 x = \frac{15}{16} - \frac{1}{16} = \frac{14}{16} = \frac{7}{8}$.

d. $\sin^2(\frac{x}{2}) = \frac{1-\cos x}{2} = \frac{1+\frac{\sqrt{15}}{4}}{2} = \frac{4+\sqrt{15}}{8}$. To choose between the positive and negative root, observe that $\frac{\pi}{2} \le x \le \pi$ implies $\frac{\pi}{4} \le \frac{x}{2} \le \frac{\pi}{2}$. So, $\frac{x}{2}$ is in Quadrant I and its sine is positive: $\sin(\frac{x}{2}) = \sqrt{\frac{4+\sqrt{15}}{8}}$.

4. (Source: 4.5.13,41, 4.7.more.3g, 4.7.more.5q, 6b)
a. Observe that $\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$. Now use the sum formula for sine:
$\sin(\frac{\pi}{4} + \frac{\pi}{3}) = \sin(\frac{\pi}{4})\cos(\frac{\pi}{3}) + \cos(\frac{\pi}{4})\sin(\frac{\pi}{3}) = \frac{1}{\sqrt{2}}\frac{1}{2} + \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$.
b. $\tan^{-1}(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$, since this is the angle in $(-\frac{\pi}{2}, \frac{\pi}{2})$ whose tangent is $-\frac{1}{\sqrt{3}}$.
c. $\frac{6\pi}{5}$ is in Quadrant III and its cosine is the x-coordinate of its terminal point. $\cos^{-1}(\cos(\frac{6\pi}{5}))$ is the angle θ in $[0, \pi]$ for which $\cos\theta = \cos(\frac{6\pi}{5})$. Since the angle between the terminal side of $\frac{6\pi}{5}$ and the x-axis is $\frac{\pi}{5}$, θ must be $\frac{4\pi}{5}$. (See the figure on the left below.)



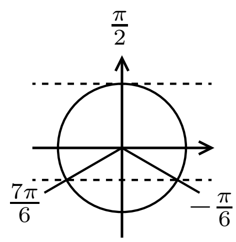


d. $\tan(\frac{6\pi}{5})$ is the slope the terminal side of $\frac{6\pi}{5}$, and $\tan^{-1}(\tan(\frac{6\pi}{5}))$ is the angle θ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ for which $\tan\theta = \tan(\frac{6\pi}{5})$. To have the same slope, the terminal sides of θ and of $\frac{6\pi}{5}$ must form a line, so θ must be $\frac{\pi}{5}$. (See the figure on right above.)

5. (Source: 4.6.more.1a) Start by rewriting the right side entirely in terms of t:

$$2\cos^2 t + \sin t - 1 = 2(1 - \sin^2 t) + \sin t - 1 = 1 + \sin t - 2\sin^2 t$$

Now factor: $0 = (1 + 2\sin t)(1 - \sin t) \Rightarrow \sin t = -\frac{1}{2}$ or $\sin t = 1$. Solutions are those t whose terminal sides intersect the unit circle at $y = -1/2$ or $y = 1$, so $t = \frac{7\pi}{6} + 2n\pi$, $-\frac{\pi}{6} + 2n\pi$, or $\frac{\pi}{2} + 2n\pi$, where $n \in \mathbb{Z}$. (See figure.)



6a. (Source: 4.9.31, 4.9.more.13) See the figure on the left below. The ladder forms the hypotenuse of a right triangle and the question asks for the vertical leg.
$\sin 70° = \text{opposite}/\text{adjacent} = \ell/20$, so $\ell = 20\sin 70°$ (which also equals $20\cos 20°$).

6b. See the figure on the right above. $\sin\theta = \text{opposite}/\text{hypotenuse} = 8/20 = 2/5$, and, as an acute angle, θ is in $(-\frac{\pi}{2}, \frac{\pi}{2})$, so $\theta = \sin^{-1}(2/5)$.

7. (Source: 4.3.34) $\csc x$ is the reciprocal of $\sin x$, so the cosecant is positive when the sine is positive, negative when the sine is positive, ± 1 when the sine is ± 1, and has a vertical asymptote when the sine has a zero. On the left is a graph of $\sin x$ and $\csc x$, and, on the right, the graph of $-2\csc x$.