No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

A mistake early in your solution does not rule out your receiving full credit for later steps.

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1 (7 pts). Solve for x: $7^{x+1} = 2^{-2x}$

2 (19 pts). Find the values of $\sin t$ and $\cos t$ at the given values of t.

t	$\pi/4$	$7\pi/2$	$7\pi/6$	$-7\pi/3$	$-4\pi/3$
$\sin t$					
$\cos t$					

3. For each of the given curves, find the equations of any horizontal or slant asymptotes, and the coordinates of any point where the curve intersects its asymptote.

a (9 pts). $y = \dfrac{(2x+1)(x-3)}{x+2}$ b (6 pts). $y = \dfrac{2x^2}{x^2+x-3}$

4 (5 pts). Find the coordinates of any holes on the graph of $y = \dfrac{2x^2-5x-3}{x^2-x-6}$.

5 (13 pts). Sketch the graph of $y = \dfrac{x+1}{x^2-4}$. Include the coordinates of all intercepts and equations of all asymptotes.

6 (6 pts). Use the properties of logarithms to break up the function until it is written entirely in terms of logs of linear functions. $\ln\left((x+2)^2 \sqrt[5]{2x-5}\right)$

7 (16 pts). Evaluate the expression.

a. $\log_3 81$ b. $\log_{25} \frac{1}{125}$ c. $\log_3 45 - \log_3 15$ d. $2^{\log_2 7}$ e. $e^{-(1/2)\ln 5}$ f. $\ln((e^3)^2))$

8 (3 pts). Covert the angle measure from degrees to radians: $20° =$

9 (16 pts). A bacterial culture contains 200 cells at time $t = 0$ and grows exponentially. At time $t = 10$ hours, the population has increased to 300 cells.

a. What was the size of the population at time $t = 6$?

b. At what time does the population contain 400 cells?

1. (Source: 5.2.more.4u) To get the variable out of the exponent, take ln of both sides, and then solve the resulting linear equation:

$$\begin{aligned} \ln(7^{x+1}) &= \ln(2^{-2x}) \\ (x+1)\ln 7 &= -2x\ln 2 \\ x\ln 7 + \ln 7 &= -2x\ln 2 \end{aligned} \qquad \begin{aligned} x\ln 7 + 2x\ln 2 &= -\ln 7 \\ x(\ln 7 + 2\ln 2) &= -\ln 7 \\ x &= \frac{-\ln 7}{\ln 7 + 2\ln 2} \end{aligned}$$

2. (Source: 4.2.more) Figure out the reference number of each angle and the quadrant containing its terminal point. Draw a vertical line from the terminal point to the x-axis and a line from the terminal point to the origin. Look for these two triangles.

t	$\pi/4$	$7\pi/2$	$7\pi/6$	$-7\pi/3$	$-4\pi/3$
Quadrant	I	South pole	III	IV	II
Reference number	$\pi/4$	$\pi/2$	$\pi/6$	$\pi/3$	$\pi/3$
$\sin t$	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos t$	$\frac{1}{\sqrt{2}}$	0	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

3a. (Source: 3.5.23-36, 3.5.more) Expand the numerator. Since the degree on top is exactly one more than the degree of the bottom, $y = \frac{2x^2-5x-3}{x+2}$ has a slant asymptote given by $y = q(x)$, the quotient in $2x^2 - 5x - 3 \div x + 2$, which we find most easily by synthetic division:

$$\begin{array}{r|rrr} -2 & 2 & -5 & -3 \\ & & -4 & 18 \\ \hline & 2 & -9 & 15 \end{array}$$

So, $y = \frac{2x^2-5x-3}{x+2} = 2x - 9 + \frac{15}{x+2}$, and $y = 2x - 9$ is a slant asymptote.

No rational function can have a graph that intersect its vertical asymptote. To see if the curve intersects its slant asymptote, set $\frac{2x^2-5x-3}{x+2} = 2x - 9$ and solve:

$$2x^2 - 5x - 3 = (2x-9)(x+2) = 2x^2 - 5x - 18 \Rightarrow -3 = -18.$$

This has no solutions, so the curve does not intersect its asymptote.

3b. The degree of the numerator and of the denomiator are equal, so the curve has a horizontal asymptote at $y = \frac{\text{lead coefficient}}{\text{lead coefficient}} = 2$. To search for an intersection, solve

$$\begin{aligned} \frac{2x^2}{x^2+x-3} &= 2 \\ 2x^2 &= 2(x^2+x-3) \\ 2x^2 &= 2x^2+2x-6 \\ 0 &= 2x-6 \\ x &= 3 \end{aligned}$$

Therefore the intersection point is $(3, 2)$.

4. (Source: 3.5.43, 3.5.more) Factor the top and bottom and look for common factors.

$$y = \frac{2x^2-5x-3}{x^2-x-6} = \frac{(2x+1)(x-3)}{(x+2)(x-3)},$$

so there's a hole at $x = 3$. The y-coordinate of the hole is the same as the y-coordinate of the curve $y = \frac{(2x+1)}{(x+2)}$ at $x = 3$, so the hole has coordinates $(3, 7/5)$.

5. (Source: 3.5.15,16) Factor: $y = \dfrac{x+1}{(x-2)(x+2)}$. Domain is all x-values except $x = \pm 2$.

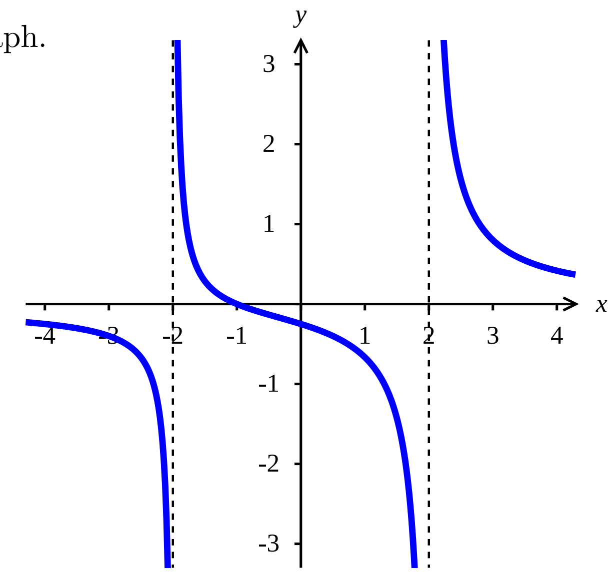
When $x = 0$, we compute $y = \frac{1}{-4}$, and when $y = 0$, we solve for x to find $0 = \frac{x+1}{(x-2)(x+2)}$, which implies $0 = x + 1$, which implies $x = -1$. So the y-intercept is $(0, -1/4)$ and the x-intercept is $(-1, 0)$.

No common factors implies no holes. Vertical Asymptotes at $x = 2$ and $x = -2$. Since degree on top is less than on bottom, $y = 0$ is a Horizontal Asymptote. Complete a sign chart to help us graph.

$$\begin{array}{rl} x-2: & - - - - - - - - - - - - - - - - 0 + + + + + + + + \\ x+1: & - - - - - - - - - 0 + + + + + + + + + + + + + \\ x+2: & - - - - - - 0 + + + + + + + + + + + + + + + + \\ \frac{x+1}{(x-2)(x+2)}: & - - - - - \text{DNE} + + + + 0 - - - - - \text{DNE} + + + + \\ \hline x: & \qquad\qquad -2 \qquad\quad -1 \qquad\qquad\quad 2 \end{array}$$

(Note that y changes sign at each zero of the numerator or denominator because these all have multiplicity one.)

5, continued. Here's a graph.



6. (Source: 5.2.51) $\ln\big((x+2)^2\sqrt[5]{2x-5}\big) = \ln(x+2)^2 + \ln(\sqrt[5]{2x-5})$
$= \ln(x+2)^2 + \ln(2x-5)^{1/5} = 2\ln(x+2) + \frac{1}{5}\ln(2x-5)$.

7. (Source: 5.2.12-22, 5.2.more.1)

a. $\log_3 81 = 4$ because $81 = 3^4$.

b. Find $\log_{25}\frac{1}{125} = x$ by writing in exponential form and solving for x:

$$25^x = \frac{1}{125} \qquad 5^{2x} = 5^{-3}$$
$$(5^2)^x = 5^{-3} \qquad 2x = -3 \Rightarrow x = -3/2$$

c. $\log_3 45 - \log_3 15 = \log_3(45/15) = \log_3 3 = \log_3 3^1 = 1$

d. $2^{\log_2 7} = 7$. ($\log_2 x$ is the exponent you raise 2 to to get x.)

e. $e^{-(1/2)\ln 5} = e^{\ln 5^{-1/2}} = 5^{-1/2}$, or $\frac{1}{\sqrt{5}}$.

f. $\ln((e^3)^2)) = \ln(e^6) = 6$

8. (Source: 4.1.17) $20°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{9}$

9. (Source: 5.3.more.2) Let y stand for the number of cells in the population at time t. Exponential growth means that $y = y_0e^{kt}$ for some constants y_0 and k. Since $y = 200$ at $t = 0$, $y_0 = 200$. Now plug in $y = 300$ and $t = 10$ and solve for k:

$$300 = 200e^{10k} \Rightarrow 3/2 = e^{10k} \Rightarrow \ln(3/2) = 10k \Rightarrow \frac{\ln(3/2)}{10} = k.$$

a. At $t = 6$, the population is $y = 200e^{\frac{6}{10}\ln(3/2)}$ (which can be simplified to $200\cdot(3/2)^{3/5}$).

b. Set $y = 400$ and solve for t:

$$400 = 200e^{t\frac{\ln(3/2)}{10}} \Rightarrow 2 = e^{t\frac{\ln(3/2)}{10}} \Rightarrow \ln 2 = t\frac{\ln(3/2)}{10} \Rightarrow \frac{10\ln 2}{\ln(3/2)} = t.$$