MATH 111–03 (Kunkle), Exam 2 Name: ______________________

100 pts, 75 minutes Feb. 23, 2016

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

A mistake early in your solution does not rule out your receiving full credit for later steps.

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1 (11 pts). A rectangular plot of land is surrounded by fence and divided into two pens by an additional length of fence parallel to one side of the rectangle. If the total area of the two pens is to be 1000 m^3, express the total length of fence as a function of the length of one side of the rectangle.

2 (10 pts). Find $f^{-1}(x)$ if $f(x) = \dfrac{1}{x^3 + 27}$. State the domain and range of f^{-1} in interval notation.

3 (14 pts). Find $g'(x)$ if $g(x) = \dfrac{x}{5 - x}$.

4 (8 pts). Expand the binomial:

a. $(u + v)^5$

b. $(s - t)^5$

5 (10 pts). Graph the equation and find all intercepts. $y = -1 + (x + 2)^3$.

6 (10 pts). Sketch the graph of $y = -3x(x + 1)^2(x - 2)^3$. Your graph needn't be to accurate scale but should show all intercepts, the graph's behavior at each, and the correct end behavior of the function.

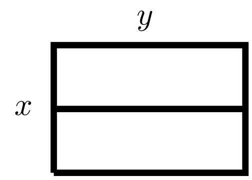
7 (15 pts). Factor completely and find all zeros of $p(x) = x^4 - 6x^3 + 13x^2 - 24x + 36$. Hint: $x = 2i$ is a zero.

8a (7 pts). Suppose that A, B, and C are integers. List all possible rational zeros of $q(x) = 3x^4 + Ax^3 + Bx^2 + Cx - 8$

8b (15 pts). Factor completely and find all zeros of $r(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$.

1 (1 pts). (Source: 2.8.29)

The total length of fence is $L = 2x + 3y$. The area of the rectangle is 1000, or $xy = 1000$. Solving, $y = 1000/x$. Plug this in for y and get $L = 2x + 3(1000/x) = 2x + 3000/x$.



(If you solve for x instead of y and wrote L in terms of y, then you'll have $L = 3y + 2000/y$. Either answer is correct.)

2 (1 pts). (Source: 2.7.more.1q) To solve for x in the equation $y = \dfrac{1}{x^3 + 27}$, start by taking the reciprocal of both sides.

$$\frac{1}{y} = x^3 + 27 \Rightarrow x^3 = y^{-1} - 27 \Rightarrow x = \sqrt[3]{y^{-1} - 27} = f^{-1}(y),$$

so $f^{-1}(x) = \sqrt[3]{x^{-1} - 27}$. (You could also write $x^{-1} - 27$ as $\frac{1-27x}{x}$ and the cube root as the 1/3rd power, but $\sqrt[3]{x^{-1} - 27} \neq \sqrt[3]{x^{-1}} - \sqrt[3]{27}$)

To find the domain of f^{-1}, note that $x^{-1} = 1/x$, so x can't equal zero. The cube root imposes no restrictions on x, so the domain of f^{-1} is $(-\infty, 0) \cup (0, \infty)$.

The range of f^{-1} is the domain of $f(x) = \frac{1}{x^3+27}$. x^3 mustn't equal -27, so we exclude $\sqrt[3]{-27} = -3$ from the domain of f. The range of f^{-1} is $(-\infty, -3) \cup (-3, \infty)$.

3 (1 pts). (Source: 2.9.21)

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h\to 0} \frac{\left(\dfrac{x+h}{5-(x+h)} - \dfrac{x}{5-x}\right)}{h} \frac{(5-x-h)(5-x)}{(5-x-h)(5-x)} \\ &= \lim_{h\to 0} \frac{(x+h)(5-x) - x(5-x-h)}{h(5-x-h)(5-x)} = \lim_{h\to 0} \frac{5x + 5h - x^2 - xh - 5x + x^2 + xh}{h(5-x-h)(5-x)} \\ &= \lim_{h\to 0} \frac{5h}{h(5-x-h)(5-x)} = \lim_{h\to 0} \frac{5}{(5-x-h)(5-x)} \\ &= \frac{5}{(5-x)(5-x)} = \frac{5}{(5-x)^2} \end{aligned}$$

4 (1 pts). (Source: Pascal.1ef) We'll need the 5th row of Pascal's triangle:

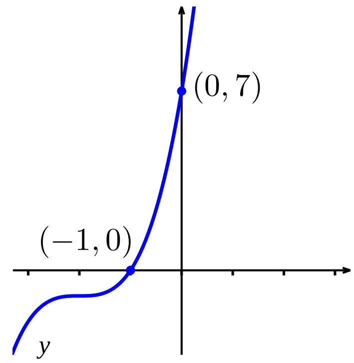
$$\begin{array}{ccccccccccc} &&&&& 1 &&&&& \\ &&&& 1 && 1 &&&& \\ &&& 1 && 2 && 1 &&& \\ && 1 && 3 && 3 && 1 && \\ & 1 && 4 && 6 && 4 && 1 & \\ 1 && 5 && 10 && 10 && 5 && 1 \end{array}$$

a. $(u+v)^5 = u^5 + 5u^4v + 10u^3v^2 + 10u^2v^3 + 5uv^4 + v^5$

b. $(s-t)^5 = (s+(-t))^5 = s^5 + 5s^4(-t) + 10s^3(-t)^2 + 10s^2(-t)^3 + 5s(-t)^4 + (-t)^5$
$= s^5 - 5s^4t + 10s^3t^2 - 10s^2t^3 + 5st^4 - t^5.$

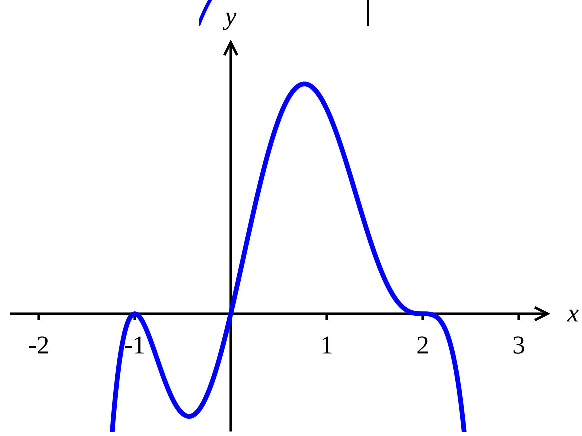
5 (1 pts). (Source: 3.1.4) Shift the familiar graph of $y = x^3$ left 2 and down 1.

To find the y-intercept, set $x = 0$ and calculate $y = -1 + (2)^3 = 7$. To find the x-intercept, set $y = 0$ and solve: $0 = -1 + (x+2)^3 \Rightarrow 1 = (x+2)^3 \Rightarrow 1 = x+2 \Rightarrow x = -1$.



6 (1 pts). (Source: 3.1.more.1b) The lead term if $-3x^6$, so $y \to -\infty$ as $x \to \pm\infty$. That is, the curve leaves the screen at the extreme bottom left and right.

$y = 0$ at $x = -1, 0, 2$, and y changes sign only at the zeros of odd multiplicity, 0 and 2. Note the flatness of the curve at $x = -1$ and 2.



7 (1 pts). (Source: 3.3.24) Since $x = 2i$ is a zero, so is its conjugate, $-2i$. Therefore $(x - 2i)(x + 2i) = x^2 + 4$ is a factor of $p(x)$. Find the other factor by long division:

$$
\begin{array}{r|l}
 & x^2 - 6x + 9 \\
x^2 + 4 & x^4 - 6x^3 + 13x^2 - 24x + 36 \\
 & -(x^4 + 4x^2) \\
 & -6x^3 + 9x^2 - 24x + 36 \\
 & -(-6x^3 - 24x) \\
 & 9x^2 + 36 \\
 & -(9x^2 + 36) \\
 & 0
\end{array}
$$

So, $p(x) = (x^2 + 4)(x^2 - 6x + 9) = (x - 2i)(x + 2i)(x - 3)^2$, and the zeros of p are $\pm 2i, 3$.

8a. (Source: 3.4.more.1m) If $x = b/a$ is a zero of $q(x)$, then b must divide the constant term 8 and a must divide the lead coefficient 3. The only possible rational zeros are

$$1,\ 2,\ 4,\ 8,\ \frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{8}{3},\ -1,\ -2,\ -4,\ -8,\ -\frac{1}{3},\ -\frac{2}{3},\ -\frac{4}{3},\ -\frac{8}{3}$$

8b. Because the lead coefficient of $r(x)$ is 1, the only possible rational zeros of $r(x)$ are

$$1,\ 2,\ 4,\ 8,\ -1,\ -2,\ -4,\ -8$$

Now we just use synthetic division to evaluate $r(x)$ at these numbers until we find a zero. With every zero we find, the degree of the remaining polynomial equation goes down by one. It turns out that the zeros are -1 (a double zero), 2, and 4. After finding two of these, factor the remaining quadratic to find the other two zeros. Here's how it looks if we start with -1 and -1:

-1	1	-4	-3	10	8
		-1	5	-2	-8
-1	1	-5	2	8	0
		-1	6	-8	
	1	-6	8	0	

So, $r(x) = (x+1)^2(x^2 - 6x + 8) = (x+1)^2(x-4)(x-2)$, and the zeros are -1, 2, and 4.