No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

A mistake early in your solution does not rule out your receiving full credit for later steps.

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1 (6 pts). Solve the inequality and write the solution set in interval notation. $|2x+5| < 3$

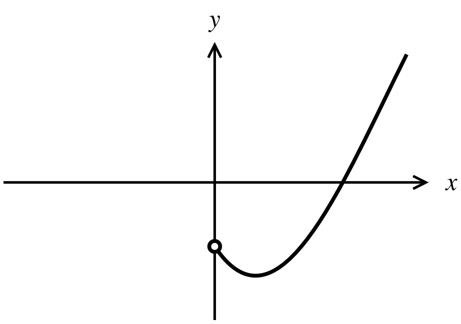
2 (10 pts). Solve the inequality and write the solution set in interval notation. $\frac{x^2-3x+4}{x+1} \leq 1$

3 (6 pts). Find the center and radius of the circle. $x^2 + 2x + y^2 - 10y - 10 = 0$

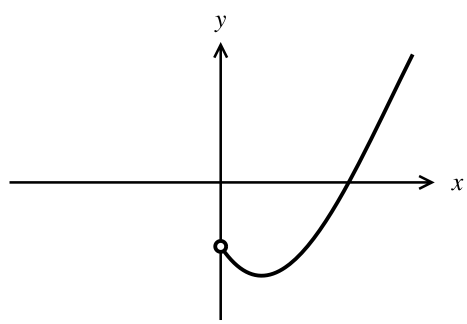
4 (5 pts). Find an equation of the line passing through the points $(3, 2)$ and $(-5, 6)$.

5 (7 pts). Evaluate the limit. $\lim_{x \to 1} \frac{(3x+1)^2 - 16}{x-1}$

6 (4 pts). Complete the graphs of the functions if $g(x)$ is even and $h(x)$ is odd.



$y = g(x)$



$y = h(x)$

7 (15 pts). Graph the equation and find all intercepts. $y = 9 - (x-1)^{2/3}$
Your graph needn't be to scale but should show the intercepts you found.

8. Let $f(x) = -2x^2 + 12x - 16$.
Work in the space below and label your answers. Recall that a quadratic function is in "standard form" when it is written in the form $u(x+v)^2 + w$ for some numbers u, v, w.

a (6 pts). Rewrite $f(x)$ in standard form.

b (15 pts). Sketch the graph of $f(x)$ on the axes below. Your drawing needn't to be to scale but should show the vertex, all intercepts, and the coordinates of all these points.

c (2 pts). Give the range of $f(x)$ in interval notation.

9. Let $f(x) = \frac{1}{x^2-9}$ and $g(x) = \sqrt{x-2}$. Find the following.

a (5 pts). The domain of $f(x)$.
b (3 pts). The domain of $g(x)$.
c (4 pts). $(fg)(x)$ and its domain.
d (5 pts). $(f/g)(x)$ and its domain.
e (7 pts). $(f \circ g)(x)$ and its domain.

Work in the space below and label your answers. State all domains in interval form. Give all functions in simplified form—no obvious simplifications undone, and no unfinished fractional algebra or arithmetic.

1. (Source: 1.2.39) $|2x+5| < 3$ means that $2x+5$ is a number less than 3 units away from 0 on the number line, so $-3 < 2x+5 < 3$. Subtract 5 from all three sides: $-8 < 2x < -2$. Dividing by 2 gives $-4 < x < -1$, so the solution set is $(-4, -1)$.

2. (Source: 1.1.55) Do not multiply both sides by $x+1$, since we don't know its sign. Instead, get zero on one side and factor the other:

$$\begin{aligned} 0 \geq \frac{x^2-3x+4}{x+1} - 1 &= \frac{x^2-3x+4}{x+1} - \frac{x+1}{x+1} = \frac{x^2-3x+4-(x+1)}{x+1} \\ &= \frac{x^2-4x+3}{x+1} = \frac{(x-1)(x-3)}{x+1} \end{aligned}$$

To decide where $(x-1)(x-3)/(x+1) \leq 0$, make a sign chart:

$$\begin{array}{rl} x-3: & ----------------0+++++++ \\ x-1: & ----------0+++++++++++++ \\ x+1: & ------0+++++++++++++++++ \\ \frac{(x-1)(x-3)}{x+1}: & -----\text{DNE}++++0-----0+++++++ \\ \hline x: & \qquad\qquad -1 \qquad\qquad 1 \qquad\qquad 3 \end{array}$$

The solution set is where the fraction is $-$ or 0. In interval form, that's $(-\infty, -1) \cup [1, 3]$.

3. (Source: 1.4.9) Complete the square:

$$\begin{aligned} x^2+2x+y^2-10y &= 10 \\ x^2+2x+1+y^2-10y+25 &= 10+1+25 \\ (x+1)^2+(y-5)^2 &= 36 \end{aligned}$$

The center is $(-1, 5)$ and radius is $\sqrt{36} = 6$.

4. (Source: 2.3.23) To find the equation of a line, we need a point on the line and the slope. We can find the slope of this line from the two points given. Remember to subtract in the same order in the top and bottom: $m = \frac{\Delta y}{\Delta x} = \frac{6-2}{-5-3} = \frac{4}{-8} = -\frac{1}{2}$.

There are now many correct answers, but if you used either of the two given points, you'd get $y-6 = -\frac{1}{2}(x+5)$, or $y-2 = -\frac{1}{2}(x-3)$. It was not necessary to write the line in slope-intercept form, but if you did, the correct answer is $y = -\frac{1}{2}x + \frac{7}{2}$.

5. (Source: 1.5.15) To evaluate the limit essentially means to simplify $\frac{(3x+1)^2-16}{x-1}$ and evaluate the result at $x = 1$. Expand the binomial $(3x+1)^2$ and collect like terms.

$$\frac{(3x+1)^2-16}{x-1} = \frac{9x^2+6x+1-16}{x-1} = \frac{9x^2+6x-15}{x-1}.$$

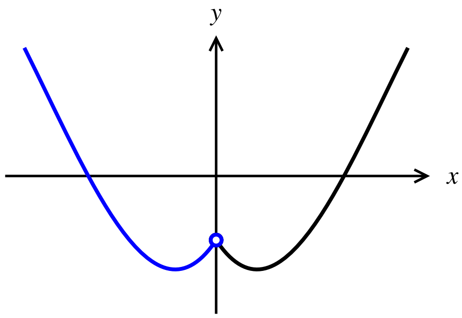
To factor the top, it pays to look for common factors first:

$$\frac{9x^2+6x-15}{x-1} = \frac{3(3x^2+2x-5)}{x-1} = \frac{3(3x+5)(x-1)}{x-1}$$

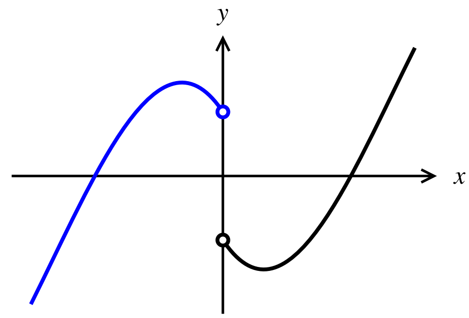
Now cancel the common factor and evaluate the result.

$$\lim_{x\to 1}\frac{3(3x+5)(x-1)}{x-1}=\lim_{x\to 1}\frac{3(3x+5)}{1}=24.$$

6. (Source: 2.2.16) The graph of g must be symmetric across the y-axis, and the graph of h must be symmetric through the origin:

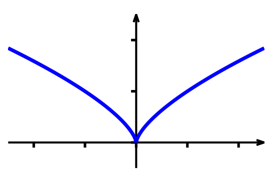


$y = g(x)$

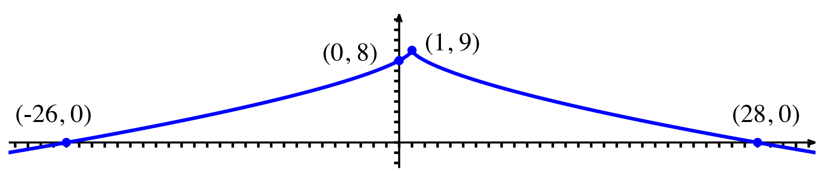


$y = h(x)$

7. (Source: 2.2.more.1) Start with the graph of $y = x^{2/3} = (\sqrt[3]{x})^2$. This looks roughly like $y = \sqrt{x}$ is quadrant 1. $\sqrt[3]{x}$ is defined for negative x-values, and when we square it, the result is positive, so the graph of $y = x^{2/3}$ looks the graph on the left:



$y = x^{2/3}$



$y = 9-(x-1)^{2/3}$

To obtain the graph of $y = 9-(x-1)^{2/3}$, reflect $y = x^{2/3}$ across the x-axis, and shift the result up 9 and to the right 1. See the graph above. To illustrate this shift, it's helpful to label $(1, 9)$ as the new "vertex."

Set $x = 0$ and calculate $y = 9-(-1)^{2/3} = 9-(\sqrt[3]{-1})^2 = 9-(-1)^2 = 9-1 = 8$. Therefore the y-intercept is $(0, 8)$.

Now set $y = 0$ solve for x.

$$\begin{aligned} 0 &= 9-(x-1)^{2/3} \\ (x-1)^{2/3} &=^* 9 \\ (x-1)^{1/3} &= \pm 9^{1/2} = \pm 3 \\ (x-1) &= (\pm 3)^3 = (\pm 1)^3 3^3 = \pm 27 \\ x &= 1 \pm 27 = -26 \text{ or } 28. \end{aligned}$$

so the x-intercepts are $(-26, 0)$ and $(28, 0)$. Students who, after the step marked *, calculated $\pm\sqrt{9^3} = \pm\sqrt{729}$ instead of $\pm(\sqrt{9})^3$ made more work for themselves than necessary.

8. (Source: 2.4.13,19)
a. Complete the square.

$$\begin{aligned} f(x) = & -2x^2 + 12x - 16 = -2(x^2 - 6x) - 16 \\ = & -2(x^2 - 6x + 9) + 18 - 16 = -2(x-3)^2 + 2. \end{aligned}$$

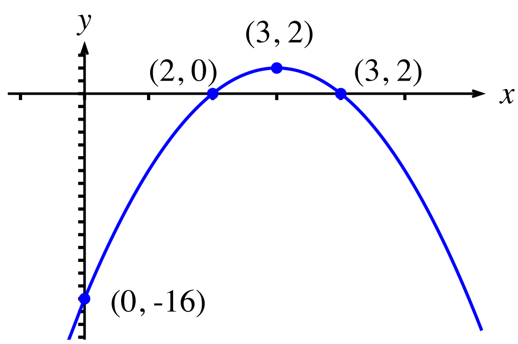
b. The vertex is at $(3, 2)$. The y-intercept is at $x = 0$, $y = f(0) = -16$. Set $y = 0$ and solve for the x-intercept. You could either use the standard form:

$$0 = -2(x-3)^2 + 2 \to 2(x-3)^2 = 2 \to (x-3)^2 = 1 \to x - 3 = \pm 1 \to x = 3 \pm 1 = 2 \text{ or } 4,$$

or factor f in its original form:

$$0 = -2x^2 + 12x - 16 = -2(x^2 - 6x + 8) = -2(x-2)(x-4) \to x = 2 \text{ or } 4.$$

The graph should open downward because $y = x^2$ has been reflected across the x-axis, and when you plot these four points, you'll find that it does:



c. All y values at or below the maximum of 2 appear on this curve, so the range is $(-\infty, 2]$.
9. (Source: 2.6.more.d) It's helpful for this problem to remember the two domain rules we've seen so far in this course:

Domain Rules

1. If a function contains $\dfrac{\blacksquare}{A}$ then it requires $A \neq 0$.
2. If a function contains $\sqrt[\text{even}]{B}$ then it requires $B \geq 0$.

a. For $f(x)$ to be defined, we can't have $x^2 = 9$, or $x = \pm 3$. Therefore, the domain of f is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
b. For $g(x)$ to be defined, we need $x - 2 \geq 0$, or $x \geq 2$. The domain of g is $[2, \infty)$.
c. $\big(fg\big)(x) = f(x)g(x) = \frac{1}{x^2-9}\sqrt{x-2}$, or $\frac{\sqrt{x-2}}{x^2-9}$. This requires both f and g to be defined, so x must be ≥ 2 and $\neq \pm 3$. This leaves $\big[2, 3\big) \cup (3, \infty)$ for the domain of fg.

d. $\big(f/g\big)(x) = \frac{1}{x^2-9} \div \sqrt{x-2} = \frac{1}{x^2-9}\frac{1}{\sqrt{x-2}} = \frac{1}{(x^2-9)\sqrt{x-2}}$ For this to be defined requires the same as in part c, as well as $\sqrt{x-2} \neq 0$, or $x \neq 2$. This leaves $(2,3) \cup (3,\infty)$ for the domain of f/g.
e. To compose f and g, use the output of g for the input of f:

$$\big(f \circ g\big)(x) = f(\sqrt{x-2}) = \frac{1}{(\sqrt{x-2})^2 - 9} = \frac{1}{x-2-9} = \frac{1}{x-11}$$

For the domain, we obviously need $x \neq 11$, but looking at $f \circ g$ in its unsimplified form, we see that it first calculates $\sqrt{x-2}$, and for this to be defined, we still need $x \geq 2$. That makes the domain $[2,11) \cup (11,\infty)$.

Most common algebra mistakes on the test

1. "$\frac{\frac{1}{a}}{b} = \frac{1}{a} \cdot \frac{b}{1}$." The reciprocal of b is $\frac{1}{b}$, not $\frac{b}{1}$, so, in fact,

$$\frac{\frac{1}{a}}{b} = \frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$$

2. "$(x \pm y)^n = x^n \pm y^n$," or "$\sqrt[n]{x \pm y} = \sqrt[n]{x} \pm \sqrt[n]{y}$." Students who make this mistake are probably thinking of the correct rule

$$(xy)^n = x^n y^n$$

3. Factoring a quadratic incorrectly when using backwards-FOIL. To test your factorization, multiply it back out and see if you get the original quadratic.
4. Not completing the square correctly. In a situation like this:

$$2x^2 + 16x - 15 = 2(x^2 + 8x + \ldots) - 15 \ldots$$

remember that whatever inside the parentheses is being multiplied by 2. In this example, when we add $(8/2)^2 = 16$ inside ( ), we must subtract $2 \cdot 16 = 32$ outside:

$$2(x^2 + 8x + 16) - 15 - 32 = 2(x+4)^2 - 47$$

5. When asked for X, answering X^2, or XY , or $\ldots$, because these are simpler than X. Your job in simplifying an expression is to change its appearance and not its actual value. If I ask for f, and you find that $f = \sqrt{2x+1}$, don't then tell me that $f = 2x+1$.
6. Mistakes with the rules of fractions. You don't need a common denominator to multiply two fractions. You can't cancel a common factor from top and bottom until the top and bottom are factored. Review the rules of arithmetic, which are, after all, the rules of algebra.