Math 111-01, Exam 2

100 pts, 50 minutes

Sept. 20, 2013

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No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers. Unless otherwise indicated, supporting work will be required on every problem; one-word answers, or answers which simply restate the question, will receive no credit.

2.5

1 (8 pts). Let \( G(x) \) be the function graphed here.

![Graph of \( G(x) \)]

Sketch the graph of \(|G(x)|\) on the axes provided.

2.4

2. Let \( f(x) = -3x^2 + 30x - 70 \).

a (12 pts). Rewrite \( f(x) \) in standard form.

b (4 pts). Find the vertex of the graph of \( f(x) \).

c (12 pts). Find all intercepts of the graph of \( f(x) \).

d (4 pts). Sketch the graph of \( f(x) \) on the axes below. Label the points you found above in parts b and c.

2.6

3. For this problem, let \( f(x) = \frac{1}{x-2} \) and \( g(x) = \frac{x-8}{x-3} \). Find and simplify the following functions and give their domains.

a (8 pts). \( (fg)(x) \)

b (12 pts). \( (f \circ g)(x) \)

2.2 more

4 (10 pts). Find the \( x \)- and \( y \)-intercepts on the curve \( y = 32 - (x - 2)^{5/4} \)

2.2 more and 2.7

5a (12 pts). Find \( \ell^{-1}(x) \) if \( \ell(x) = (x - 2)^{-5/3} + 32 \)

5b (6 pts). Find the domain and range of \( \ell(x) \). Write your answers in interval notation, and label them so I can tell which is which.
6 (12 pts). Identify each function shown as either one-to-one or not one-to-one. For each one-to-one function you find, sketch the graph of the inverse function on the same axes.

- [ ] one-to-one
- [ ] not one-to-one

- [ ] one-to-one
- [ ] not one-to-one

- [ ] one-to-one
- [ ] not one-to-one
1. \( y = |G(x)| \)

2. a. \( f(x) = -3x^2 + 30x - 70 \)
   \[-\frac{1}{3} f(x) = x^2 - 10x + \frac{70}{3} = x^2 - 10x + 25 - 25 + \frac{70}{3} \]
   \[-\frac{1}{3} f(x) = (x-5)^2 - 25 + \frac{70}{3} \]
   \[f(x) = -3(x-5)^2 + 75 - 70 = -3(x-5)^2 + 5 \]

   b. vertex: \((5, 5)\)

   c. \( x=0 \Rightarrow y = -70 \)  
      \( y=0 \Rightarrow 0 = -3(x-5)^2 + 5 \)  
      \[3(x-5)^2 = 5 \]  
      \[(x-5)^2 = \frac{5}{3} \]
      \[ x-5 = \pm \sqrt{\frac{5}{3}} \]  
      \[ x = 5 \pm \sqrt{\frac{5}{3}} \]

2. a. \( f(x) = -3x^2 + 20x - 68 \)  
   \[-\frac{1}{3} f(x) = x^2 - 10x + \frac{68}{3} = x^2 - 10x + 25 - 25 + \frac{68}{3} \]
   \[-\frac{1}{3} f(x) = (x-5)^2 - 25 + \frac{68}{3} \]
   \[f(x) = -3(x-5)^2 + 75 - 68 = -3(x-5)^2 + 7 \]

   b. vertex is \((5, 7)\)

   c. \( x=0 \Rightarrow y = -68 \)  
      \( y=0 \Rightarrow 0 = -3(x-5)^2 + 7 \)  
      \[3(x-5)^2 = 7 \]  
      \[(x-5)^2 = \frac{7}{3} \]  
      \[ x-5 = \pm \sqrt{\frac{7}{3}} \]  
      \[ x = 5 \pm \sqrt{\frac{7}{3}} \]
3. a. \( (fg)(x) = \frac{1}{x^2} \cdot \frac{x-5}{x-3} = \frac{x-5}{x^2(x-3)} \quad \text{Domain} = (-\infty, 3) \cup (3, \infty) \)

b. \( (fog)(x) = f(g(x)) = f\left(\frac{x-5}{x-3}\right) = \frac{1}{\frac{x-5}{x-3} - 2} \cdot \frac{x-3}{x-3} = \frac{x-3}{-2 - x} = \frac{x-3}{-2 - x} \)

\( \text{Domain. See } \star \)

need \( x-3 \neq 0 \) and \( \frac{x-5}{x-3} - 2 \neq 0 \); \( x-8-2(x-3) \neq 0 \)

\( x \neq 3 \)

\( -x + 2 \neq 0 \); \( x \neq 2 \)

\( \text{Domain} = (-\infty, 3) \cup (3, \infty) \)

4. \( x=0 \Rightarrow y = 32 - (-2)^{\frac{5}{4}} \). DNE, can't take 4th root of -2.

\( y=0 \Rightarrow 0 = 32 - (x-2)^{\frac{5}{4}} \); \( (x-2)^{\frac{5}{4}} = 32 \)

\( x-2 = 32^{\frac{4}{5}} = (5^{\frac{4}{5}})^4 = 2^4 = 16 \).

\( x = 2 + 16 = 18 \).

5a. \( y = (x-2)^{-\frac{5}{4}} + 32 \)

\( y-32 = (x-2)^{-\frac{5}{4}} \)

\( -\frac{3}{4} \)

\( (y-32) = (x-2)^{-\frac{3}{4}} \)

\( 2 + (y-32)^{\frac{3}{4}} = x \)

\( \phi'(x) = 2 + (x-32)^{-\frac{3}{4}} \).

5b. Can't raise 0 to negative exponent.

\( \text{Domain } l(x) \text{ is all } x \text{ except } 2: \)

\( (-\infty, 2) \cup (2, \infty) \).

\( \text{Range of } l = \text{Domain } l' = (-\infty, 32) \cup (32, \infty) \).

6. 

\( \square \) one-to-one

\( \not\square \) not one-to-one

\( \square \) one-to-one

\( \not\square \) not one-to-one

\( \not\square \) not one-to-one

Alternate form: order of pictures was different.

Extra credit on this exam; problem 3b was worth 13 points instead of 12.