No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers. Guessing the correct answer is not sufficient for full credit. Supporting work will be required on every problem unless otherwise indicated.

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1 (4 pts). Evaluate each expression at $x = -1$ and $y = 3$.

a. $3x - 2y$ b. $|3x - 2y| - |2y - 1|$ c. $\sqrt{(3x)^2}$

2 (4 pts). Evaluate each expression.

a. $-2 + 18 \cdot 3^{-2} - \left(\frac{1}{2} \cdot 2 - 3\right)^3$ b. $\frac{-\frac{6}{7}}{4}$

3 (6 pts). Simplify the expression. Express your answer so that all exponents are positive. (Assume that no variables equal 0.)

$\left(\frac{(a^{3/2}bc^{-1})^4}{(c^{-3}b^{2/3}a^3)^2}\right)^{1/2}$

4 (9 pts). Find the product. Express your answer as a single polynomial in standard form.

a. $(2x + 5)(x - 3)$ b. $(2 - 5x)(2 + 5x)$ c. $(3x - 1)^2$

5 (21 pts). Factor each polynomial completely.

a. $x^5 - 16x^3$ b. $4x^2 + 4x + 1$ c. $9x^2 - 1$

d. $8x^3 + 27$ e. $6x^2 - x - 12$ f. $2x^2(3x - 4) + (3x - 4)$

6a (9 pts). Find the quotient and remainder in $(2x^4 - x^3 + x + 5) \div (x^2 - 2x + 2)$. (Label your answers so I can tell which is which.)

6b (1 pts). Based on your answer to part 6a above, would you say that $x^2 - 2x + 2$ a factor of $2x^4 - x^3 + x + 5$?

7 (5 pts). The width of a rectangle is 2 feet less than its length, and its perimeter is 14 feet. Find the width and length of the rectangle.

8 (18 pts). Perform the indicated operation and simplify the result. Leave your answer in factored form.

a. $\frac{x+5}{(x+7)^2} \div \frac{x^2+4x-5}{x^2-49}$ b. $\frac{1}{x^2-1} - \frac{3}{x^2+x}$ c. $\frac{4+\frac{1}{3x+1}}{2-\frac{x}{3x+1}}$

9 (10 pts). Simplify each expression. Assume all variables are positive.

a. $4^{-1/2}$ b. $16^{3/4}$ c. $(\sqrt{5} + 4)(\sqrt{5} - 1)$ d. $\sqrt{50x^5}$

10 (4 pts). Test the graph of $x^2 = y^3 - 2y^2$ for symmetry with respect to the x-axis and state your conclusions.

11 (7 pts). Solve each equation.

a. $6 - 3x = 2x$ b. $5x + 4 = 2x - 3(1 - x)$. c. $\frac{4}{x+4} + \frac{3}{x-1} = 0$

12 (4 pts). Rewrite $(3 + i)(5 - 4i)$ in the form $a + bi$.

13 (12 pts). Find all real solutions.

a. $2x^2 - 13x + 15 = 0$ b. $(x + 2)^2 = 32$. c. $4x^2 - 2 = x$

14 (20 pts). Find all real solutions.

a. $\sqrt{14 - x} = 2 - x$ b. $3x^3 = 5x^2$ c. $x^4 + 3x^2 - 10 = 0$

15 (5 pts). Find all solutions in the complex number system: $x^2 - 4x + 6 = 0$

16 (14 pts). Solve each inequality. Express your answer in interval notation.

a. $-\frac{1}{4}x < 3$ b. $1 - x \geq \frac{1}{9}(2x + 1)$ c. $|2 + 3x| \leq 5$

17 (6 pts). Find all real solutions.

a. $|1 + 2x| = 5$ b. $|3x - 2| = -7$

18. My car gets 30 miles per gallon of gas in the city and 40 miles per gallon on the highway. Answer the following **two** questions based on this mileage.

a (2 pts). If I drive x miles in the city, how many gallons of gas will I use?
(The answer is an expression involving x.)

b (10 pts). This weekend I drove 111 miles and used 3 gallons of gas. How many miles did I drive in in the city and how many on the highway?
(Label your answers so I can tell which is which.)

19 (6 pts). Determine whether or not the triangle ABC is a right triangle if A, B, and C are the three points $(1, 0)$, $(1, 7)$, and $(4, 2)$. Briefly explain how you come to your conclusion.

20 (3 pts). Find an equation of the circle with radius 3 centered at the point $(-4, 2)$. (Supporting work not required on this problem.)

21 (7 pts). Find the center and radius of the circle $x^2 + 8x + y^2 - 4y = -2$.

22 (8 pts). Find an equation of the line described in each part.

a. The line passing through the two points $(1, 0)$ and $(5, -3)$.

b. The line passing through the point $(8, -9)$ and parallel to the line $y = 3x + 17$.

23 (5 pts). Find all intercepts and graph the equation on the axes provided.

a. $2x - 3y = 6$ b. $2y = 3$

1a. $3x-2y = 3(-1)-2(3) = -3-6 = -9.$

b. $|3x-2y| - |2y-1| = |-9| - |2\cdot 3-1| = 9-|6-1| = 9-|5| = 4.$

c. $\sqrt{(3(-1))^2} = \sqrt{(-3)^2} = \sqrt{9} = 3.$

2a. $-2+18\cdot 3^{-2} - (\frac{1}{2}\cdot 2-3)^3 = -2+18\cdot\frac{1}{3^2} - (1-3)^3$

$= -2+18\cdot\frac{1}{9} - (-2)^3 = -2+2-(-2)(-2)(-2) = -(-8) = 8.$

2b. $-\frac{6}{7}\div 4 = -\frac{6}{7}\cdot\frac{1}{4} = \frac{-3\cdot 2}{7\cdot 2\cdot 2} = \frac{-3}{7\cdot 2} = \frac{-3}{14}.$

3. $\dfrac{(a^{3/2}bc^{-1})^{\overbrace{4\cdot\frac{1}{2}}^{=2}}}{(c^{-3}b^{2/3}a^3)^{\underbrace{2\cdot\frac{1}{2}}_{=1}}} = \dfrac{a^{3/2\cdot 2}b^{2}c^{-1\cdot 2}}{c^{-3}b^{2/3}a^3} = \dfrac{a^3b^2c^{-2}}{c^{-3}b^{2/3}a^3} = \dfrac{b^{2-2/3}c^{-2-(-3)}}{1}$

$= b^{4/3}c.$

4a. $(2x+5)(x-3) = \underset{F}{2x^2}\ \underset{O}{-6x}\ \underset{I}{+5x}\ \underset{L}{-15} = 2x^2-x-15.$

b. $(2-5x)(2+5x) = 2^2-(5x)^2 = 4-25x^2$

$\boxed{(a-b)(a+b) = a^2-b^2}$ $\boxed{(a-b)^2 = a^2-2ab+b^2}$

c. $(3x-1)^2 = (3x)^2-2\cdot 1\cdot 3x+1^2 = 9x^2-6x+1.$

5a. $x^3(x^2-16) = x^3(x-4)(x+4)$

b. $4x^2+4x+1 = (2x+1)^2$ c. $9x^2-1 = (3x-1)(3x+1)$

d. $8x^3+27 = (2x)^3+3^3 = (2x+3)((2x)^2-(2x)\cdot 3+3^2)$

$= (2x+3)(4x^2-6x+9).$

e. $6x^2-x-12 = (2x-3)(3x+4).$

f. $(2x^2+1)(3x-4).$

6a.
$$
\begin{array}{r|l}
 & \quad\ \ 2x^2 + 3x + 2 \\ \hline
x^2-2x+2 & 2x^4 - x^3 + 0x^2 + x + 5 \\
 & -(2x^4 - 4x^3 + 4x^2) \\ \cline{2-2}
 & \quad 3x^3 - 4x^2 + x + 5 \\
 & \quad -(3x^3 - 6x^2 + 6x) \\ \cline{2-2}
 & \qquad 2x^2 - 5x + 5 \\
 & \qquad -(2x^2 - 4x + 4) \\ \cline{2-2}
 & \qquad\qquad -x + 1
\end{array}
$$

$Q = 2x^2+3x+2$

$R = -x+1$

6b. No (because $R \neq 0$)

7. Let w = width of rectangle, in feet, and l = length of rect., in ft.

$w = l-2$. perimeter $= 2w+2l = 14$

$w + l = 7$

$(l-2)+l = 7$; $2l-2=7$; $2l = 9$; $l = \frac{9}{2}$. $w = \frac{9}{2}-2 = \frac{5}{2}$.

8a. $\dfrac{x+5}{(x+7)^2} \cdot \dfrac{x^2-49}{x^2+4x-5} = \dfrac{(x+5)(x-7)(x+7)}{(x+7)^2(x+5)(x-1)} = \dfrac{(x-7)}{(x+7)(x-1)}$.

8b. $\dfrac{1}{(x-1)(x+1)} - \dfrac{3}{x(x+1)} = \dfrac{x}{x} \cdot \dfrac{1}{(x-1)(x+1)} - \dfrac{3}{x(x+1)} \cdot \dfrac{(x-1)}{(x-1)}$

$= \dfrac{x-3(x-1)}{x(x-1)(x+1)} = \dfrac{x-3x+3}{''} = \dfrac{-2x+3}{x(x-1)(x+1)}$.

8c. $\dfrac{4+\frac{1}{3x+1}}{2-\frac{x}{3x+1}} \cdot \dfrac{(3x+1)}{(3x+1)} = \dfrac{4(3x+1)+1}{2(3x+1)-x} = \dfrac{12x+4+1}{6x+2-x} = \dfrac{12x+5}{5x+2}$.

9a. $4^{-1/2} = \dfrac{1}{4^{1/2}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$. b. $16^{3/4} = (\sqrt[4]{16})^3 = (\sqrt[4]{2^4})^3 = 2^3 = 8$.

c. $(\sqrt{5}+4)(\sqrt{5}-1) = (\sqrt{5})^2 - \sqrt{5} + 4\sqrt{5} - 4 = 5 + 3\sqrt{5} - 4 = 1 + 3\sqrt{5}$.

d. $\sqrt{2 \cdot 5^2 x^4} = (2 \cdot 5^2 \cdot x^4)^{1/2} = 2^{1/2} 5^{2 \cdot \frac{1}{2}} x^{4 \cdot \frac{1}{2}} = 2^{1/2} 5^1 x^2 = 5x^2\sqrt{2}$.

10. Replace y w/ $-y$. Result : $x^2 = (-y)^3 - 2(-y)^2$,

$x^2 = (-1)^3 y^3 - 2(-1)^2 y^2$

$x^2 = -y^3 - 2y^2$

This does not simplify to the original $x^2 = y^3 - 2y^2$, so curve is not symmetric with respect to x-axis.

11a. $6-3x = 2x \xrightarrow{\text{add } 3x} 6 = 5x \xrightarrow{\div 5} \frac{6}{5} = x$.

b. $5x+4 = 2x-3+3x$; $5x+4 = 5x-3$; $4 = -3$. NO SOLUTIONS

subtract $5x$

c. $\dfrac{4}{x+4} = -\dfrac{3}{x-1}$; $(x+4)(x-1) \cdot \dfrac{4}{x+4} = -\dfrac{3}{(x-1)}(x+4)(x-1)$

$4(x-1) = -3(x+4)$; $4x-4 = -3x-12$; $7x = -8$; $x = \frac{-8}{7}$.

12 $(3+i)(5-4i) = 3\cdot 5 - 3\cdot 4i + 5i - 4i^2$

$= 15 - 12i + 5i - 4(-1) = 15 - 7i + 4 = 19 - 7i.$

13a. $(2x-3)(x-5)=0$; $2x-3=0$ OR $x-5=0.$

$x = 3/2$ OR $x=5.$

b. $(x+2)^2 = 32 \Rightarrow x+2 = \pm\sqrt{32} = \pm\sqrt{16}\sqrt{2} = \pm 4\sqrt{2}.$

$\Rightarrow x = -2 \pm 4\sqrt{2}$ (i.e., $x = -2+4\sqrt{2}$ or $-2-4\sqrt{2}$.)

c. $4x^2 - x - 2 = 0$. Can't factor left side, so use Quad. formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{1-4\cdot 4(-2)}}{2\cdot 4} = \frac{1 \pm \sqrt{33}}{8}.$$

14a. $\boxed{\sqrt{14-x} = 2-x}$ ★ ; $(\sqrt{14-x})^2 = (2-x)^2$; $14-x = 4-4x+x^2$;

$0 = x^2-3x-10 = (x-5)(x+2) \Rightarrow x = 5$ or -2. Because we squared both sides, must check these in ★;

$\sqrt{14-5} = 2-5$; $\sqrt{9} = -3$ FALSE.

$\sqrt{14-(-2)} = 2-(-2)$; $\sqrt{16} = 4$ TRUE. Conclusion: $\underline{x=-2}$.

14b $3x^3 - 5x^2 = 0$; $x^2(3x-5) = 0 \Rightarrow x^2=0$ or $3x-5=0$

$\Rightarrow x=0$ OR $x = 5/3.$

14c. $(x^2+5)(x^2-2) = 0 \Rightarrow x^2+5=0$ OR $x^2-2=0$

$\Rightarrow x^2=-5$ OR $x^2 = 2$

no real solutions $\quad \Rightarrow x = \pm\sqrt{2}.$

(would also accept $x = \pm i\sqrt{5}, \pm\sqrt{2}$)

15. Left side doesn't factor. Quad. formula:

$$x = \frac{4 \pm \sqrt{16-4\cdot 1\cdot 6}}{2} = \frac{4\pm\sqrt{-8}}{2} = \frac{4 \pm \sqrt{-1\cdot 4\cdot 2}}{2} = \frac{4 \pm i2\sqrt{2}}{2}$$

$$= \frac{2(2 \pm i\sqrt{2})}{2} = 2 \pm i\sqrt{2}.$$

16 a. $-\frac{1}{4}x < 3 \Rightarrow (-4)(-\frac{1}{4}x) > -4\cdot 3$; $x > -12$. $(-12, \infty)$.

b. Multiply both sides by 9: $9(1-x) \geq (2x+1)$; $9-9x \geq 2x+1$;

$9-1 \geq 2x+9x$; $8 \geq 11x$; $\frac{8}{11} \geq x$. $(-\infty, \frac{8}{11}]$.

c. Rule: $|A| \leq B \Rightarrow -B \leq A \leq B$. (if $B \geq 0$)

$|2+3x| \leq 5 \Rightarrow -5 \leq 2+3x \leq 5$; $-5-2 \leq 3x \leq 5-2$;

$-7 \leq 3x \leq 3 \Rightarrow -\frac{7}{3} \leq x \leq 1$. $[-\frac{7}{3}, 1]$.

17a. Rule; If $B \geq 0$, $|A| = B \Rightarrow A = \pm B$.

$|1+2x| = 5 \Rightarrow 1+2x = \pm 5$; $2x = -1 \pm 5$; $x = \frac{-1\pm 5}{2}$,

or $x = \frac{4}{2} = 2$ or $\frac{-6}{2} = -3$.

b. $|3x-2| = -7$ has no solutions, since absolute value cannot $= -7$ (or any number < 0, for that matter).

18a. x miles $\cdot \frac{1 \text{ gallon}}{30 \text{ miles}} = \frac{1}{30}x$ gallons.

b. Let x = distance driven in city in miles, and

y = " " on highway.

Total miles: $x+y = 111$. Total gas: $\frac{1}{30}x + \frac{1}{40}y = 3$

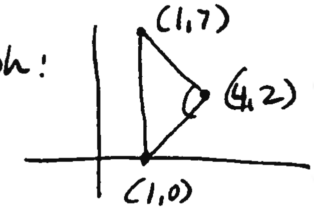
$y = 111-x$; $\frac{1}{30}x + \frac{1}{40}(111-x) = 3$. Multiply by 120.

$4x + 3(111-x) = 360$; $4x + 333 - 3x = 360$; $x + 333 = 360$

$x = 360-333 = 27$. $y = 111-27 = 84$.

Ans: I drove 27 mi. in city, 84 on highway.

19. Rough graph:



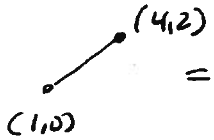
Solution one: Find 2 slopes:

$\frac{2-0}{4-1} = \frac{2}{3}$; $\frac{7-2}{1-4} = \frac{5}{-3}$.

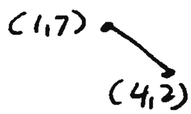
Not negative reciprocals, so $\angle$ not a right angle.

Solution two: Find 3 sides.

$= 7$.



$= \sqrt{(4-1)^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13}$



$= \sqrt{(7-2)^2 + (4-1)^2} = \sqrt{25+9} = \sqrt{34}$.

Check pythagorean theorem: $7^2 \stackrel{?}{=} \sqrt{13}^2 + \sqrt{34}^2$

$49 \stackrel{?}{=} 13 + 34 = 47$. No.

Conclusion: not a right triangle.

20. $(x+4)^2 + (y-2)^2 = 9$

21. $x^2 + 8x + y^2 - 4y = -2 \Rightarrow x^2 + 8x + 16 + y^2 - 4y + 4 = -2 + 16 + 4$;

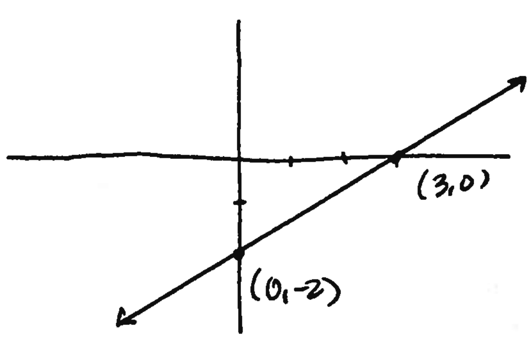
$(x+4)^2 + (y-2)^2 = 18$ center $= (-4, 2)$. radius $= \sqrt{18} = 3\sqrt{2}$

22a. $m = \frac{-3-0}{5-1} = -\frac{3}{4}$. $y - 0 = -\frac{3}{4}(x-1)$ (or $y + 3 = -\frac{3}{4}(x-5)$)

b. $y = 3x + 17$ has slope 3, so our line also has slope 3:

$y + 9 = 3(x-8)$.

23a. $x = 0 \Rightarrow -3y = 6 \Rightarrow y = -2$
$y = 0 \Rightarrow 2x = 6 \Rightarrow x = 3$



23b. $2y = 3 \Rightarrow y = 3/2$.
horizontal line at altitude 3/2. NO x-intercept.
y-intercept is (0, 3/2):