Math 101–01 (Kunkle), Exam 2 Name: ______________________

150 pts, 75 minutes May 28, 2013 

No notes, books, electronic devices, or outside materials of any kind.

Question is from section: Read each problem carefully and simplify your answers. Unless otherwise indicated, supporting work will be required on every problem; one-word answers will not receive full credit.

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3.6 1 (9 pts). A wire of length $x$ is bent into a circle. Express the area of the circle as a function of $x$.

1.6 2 (10 pts). Find the real solutions (if any) of each equation.

a. $|\frac{3}{5}x - \frac{1}{3}| = 1$ b. $|x-2| = -4$

1.3 (a) 3. Find the real and complex solutions (if any) of each equation.

1.4 (b) a (10 pts). $x^2 + 6x + 25 = 0$ b (12 pts). $x + 2 = \sqrt{3x+10}$

1.5 (a) 4. Solve for each inequality. Express your answer using interval notation.

1.6 (b) a (9 pts). $-2(x-2) < 5(x-5) + 3$ b (9 pts). $|4 - 3x| \leq 3$.

2.1 $\left\{ \begin{array}{l} \text{5 (3 pts). Find the midpoint of the line segment joining the points } (2,4) \text{ and } (1,-9). \\ \text{6 (3 pts). Find the distance between the points } (2,4) \text{ and } (1,7). \end{array} \right.$

2.4 7 (6 pts). Find an equation of the circle centered at $(3,-2)$ and containing the point $(4,-1)$.

3.1 8 (8 pts). Find the domain of $\frac{\sqrt{x+3}}{2x-20}$. Express your answer in interval notation.

9 (15 pts). Find an equation of the line described in each part.

2.3 a. The line passing through the two points $(-1,3)$ and $(5,7)$.

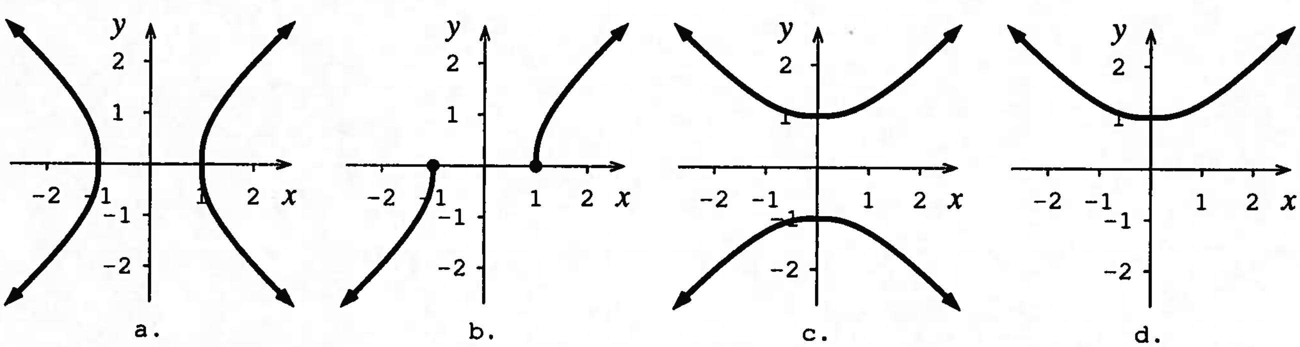
b. The line passing through the point $(-1,3)$ parallel to the line $2x + 3y = 1$.

c. The line passing through the point $(-1,3)$ perpendicular to the line $2x + 3y = 1$.

2.3 (a,b) 10 (12 pts). Graph the functions on the axes provided.

3.4 (c) a. $y = 2$ b. $y = -\frac{1}{2}x + 1$ c. $y = \begin{cases} -\frac{1}{2}x + 1 & \text{if } x < -2 \\ x & \text{if } x \geq -2 \end{cases}$

3.2 11 (10 pts). Check the box to indicate whether or not each of the graphs below is that of a function. For each function, state its domain and range in interval notation.




a. ☐ *not* a function. ☐ function; domain = ______________ range = ______________

b. ☐ *not* a function. ☐ function; domain = ______________ range = ______________

c. ☐ *not* a function. ☐ function; domain = ______________ range = ______________

d. ☐ *not* a function. ☐ function; domain = ______________ range = ______________

12. This problem is about the function $g(x) = 2x^2 - 8x + 7$ and its graph.

4.3

a (10 pts). Write $g(x)$ in the form $a(x-h)^2 + k$.

b (2 pts). Find the coordinates of the vertex of the graph.

c (2 pts). Find the equation of the axis of symmetry of the graph.

4.3, 3.2 2.2

d (10 pts). Find the $x$-intercepts, if any, of the graph.

e (2 pts). Find the $y$-intercepts, if any, of the graph.

4.3

f (2 pts). Write the range of $g(x)$ in interval form.

g (6 pts). Sketch the graph of $g(x)$ on the axes below. Your graph need not be to scale but should include the features you found in parts a-f.

You won't be provided this information on your exams

1. Area $= \pi r^2$. Circumf. $= 2\pi r = x \Rightarrow r = \frac{x}{2\pi} \Rightarrow A = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{\pi x^2}{4\pi^2} = \frac{x^2}{4\pi}$.

2a. $\left|\frac{3}{5}x - \frac{1}{3}\right| = 1 \Rightarrow \frac{3}{5}x - \frac{1}{3} = \pm 1$. will solve each of these.

$15\left(\frac{3}{5}x - \frac{1}{3}\right) = 1 \cdot 15 \Rightarrow 9x - 5 = 15 \Rightarrow 9x = 20 \Rightarrow x = \frac{20}{9}$.

$15\left(\frac{3}{5}x - \frac{1}{3}\right) = -1 \cdot 15 \Rightarrow 9x - 5 = -15 \Rightarrow 9x = -10 \Rightarrow x = -10/9$.

2b. $|x-2| = -4$ has NO SOLUTIONS. (abs. value can't be negative.)

3a. $x^2 + 6x + 25 = 0$. Can't be factored. Use Quadratic Formula.

$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 25}}{2} = \frac{-6 \pm \sqrt{-64}}{2} = \frac{-6 \pm 8i}{2} = \frac{2(-3 \pm 4i)}{2} = -3 \pm 4i$

alt. form: replace 6 w/ −6. $x = +3 \pm 4i$.

3b Square both sides. $(x+2)^2 = 3x + 10 \Rightarrow x^2 + 4x + 4 = 3x + 10 \Rightarrow x^2 + x - 6 = 0$.

$(x+3)(x-2) = 0 \Rightarrow x = -3$ or $+2$. Check in original equation:

$-3 + 2 = -1 \neq \sqrt{\text{anything}}$. $2 + 2 = 4 = \sqrt{3 \cdot 2 + 10} = \sqrt{16}$ ✓. $x = 2$ is only solution.

alt form: $x + 2 = \sqrt{3x + 16}$. Square: $x^2 + 4x + 4 = 3x + 16 \Rightarrow$

$x^2 + x - 12 = 0 \Rightarrow (x+4)(x-3) = 0 \Rightarrow x = -4, 3$. Check in original.

$-4 + 2 = -2 \neq \sqrt{\text{anything}}$. $3 + 2 = \sqrt{3 \cdot 3 + 16} = \sqrt{25}$ ✓. $x = 3$ is only solution.

4a. $-2x + 4 < 5x - 25 + 3 = 5x - 22$. Add $2x + 22$: $26 < 7x \Rightarrow \frac{26}{7} < x$.

Sol'n set $= \left(\frac{26}{7}, \infty\right)$.

alt. $-2x + 4 < 5x - 25 - 3 = 5x - 28$. Add $2x + 28$. $32 < 7x \Rightarrow \frac{32}{7} < x$.

Sol'n set $= \left(\frac{32}{7}, \infty\right)$.

4b. $|4-3x| \le 3. \Rightarrow \quad -3 \le 4-3x \le 3$ ; $-7 \le -3x \le -1$; $\frac{-7}{-3} \ge x \ge \frac{-1}{-3}$, or

$\frac{1}{3} \le x \le \frac{7}{3}$. Solution set $= [1/3, 7/3]$.

5. midpt $= \left(\frac{2+1}{2}, \frac{4+(-9)}{2}\right) = \left(\frac{3}{2}, \frac{-5}{2}\right)$. Alt: midpt $= \left(\frac{-2+1}{2}, \frac{4+9}{2}\right) = \left(\frac{-1}{2}, \frac{13}{2}\right)$.

6. dist $= \sqrt{(2-1)^2+(4-7)^2} = \sqrt{10}$. alt dist $= \sqrt{(2-1)^2+(4-8)^2} = \sqrt{17}$.

7. radius = dist $(3,-2)$ to $(4,-1)$ $= \sqrt{(3-4)^2+(-2+1)^2} = \sqrt{2} = r$. $r^2=2$.

circle : $(x-3)^2+(y+2)^2=2$.

8. Function needs $x+3 \ge 0$ and $2x-20 \ne 0$; $x \ge -3$ and $x \ne 10$.

Domain $= [-3,10) \cup (10,\infty)$.

Alt. $x+3 \ge 0$ and $2x-18 \ne 0$. $[-3,9) \cup (9,\infty)$.

9. a. slope $= (7-3)/(5-(-1)) = 4/6 = 2/3$. line is $y-3 = \frac{2}{3}(x+1)$

b. Give line: $2x+3y=1$ ; $3y = 1-2x$ ; $y = -\frac{2}{3}x + \frac{1}{3}$. slope $= -2/3$.
line in question is $y-3 = -\frac{2}{3}(x+1)$

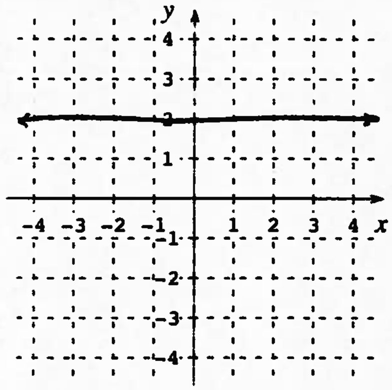
c. given line has slope $-2/3$. Desired line has slope $+3/2$.
line is $y-3 = \frac{3}{2}(x+1)$.

Alt. a. slope $= (5-3)/(7+1) = 2/8 = 1/4$. ans is $y-3 = \frac{1}{4}(x+1)$

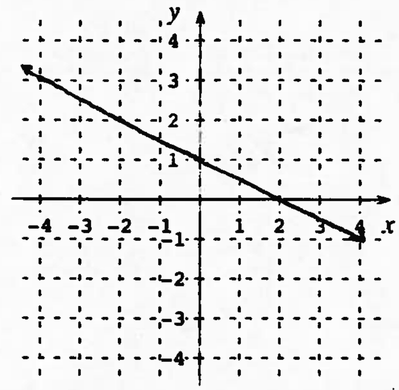
b. $2x-3y=1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3}$ has slope $\frac{2}{3}$. ans $y-3 = \frac{2}{3}(x+1)$

c. Since $2x-3y=1$ has slope $\frac{2}{3}$, line in question has slope $-3/2$. ans: $y-3 = -\frac{3}{2}(x+1)$.

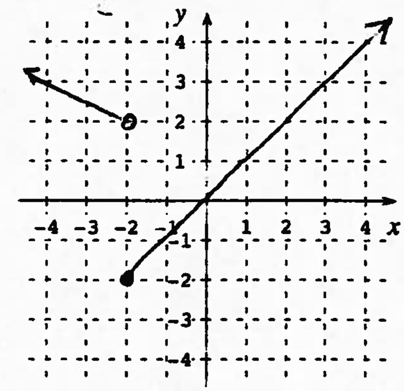
10. $y = 2$



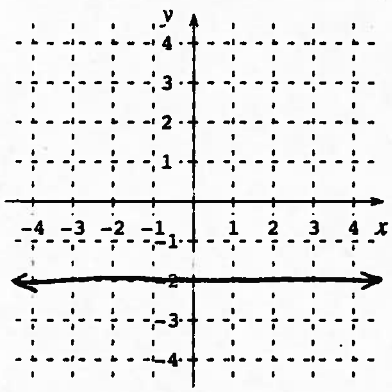
$y = -\frac{1}{2}x + 1$



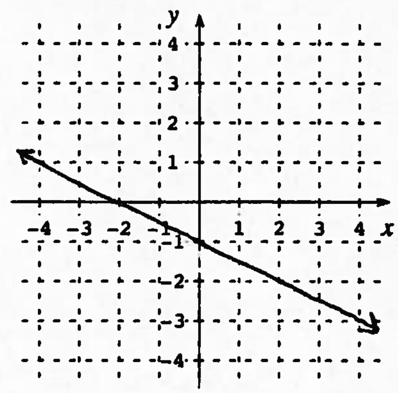
$y = \begin{cases} -\frac{1}{2}x + 1 & x < -2 \\ x & x \geq -2 \end{cases}$



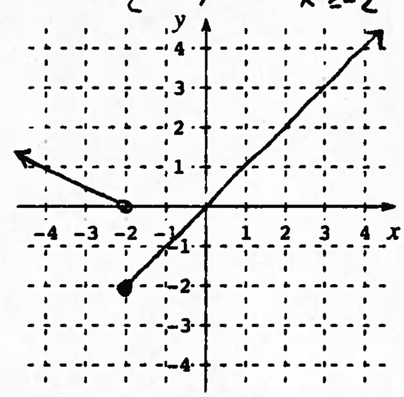
alt: $y = -2$



$y = \frac{-1}{2}x - 1$



$y = \begin{cases} -\frac{1}{2}x - 1 & x < -2 \\ x & x \geq -2 \end{cases}$



11a. Not a function. b. Function; domain = $(-\infty, -1] \cup [1, \infty)$, range = $(-\infty, \infty)$

c. Not a function d. Function; domain = $(-\infty, \infty)$, range = $[1, \infty)$

alt: a. Function; dom = $(-\infty, -1] \cup [1, \infty)$, range = $(-\infty, \infty)$. b. Not a function

c. Function; domain = $(-\infty, \infty)$, range = $[1, \infty)$. d. Not a function.

12. a. $\frac{1}{2}g(x) = x^2 - 4x + \frac{7}{2} = x^2 - 4x + 4 - 4 + \frac{7}{2} = (x-2)^2 - 4 + \frac{7}{2}$.

$\Rightarrow g(x) = 2(x-2)^2 - 8 + 7 = 2(x-2)^2 - 1$.

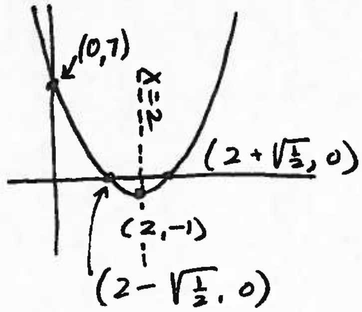
b. vertex = $(2, -1)$ c. $x = 2$.

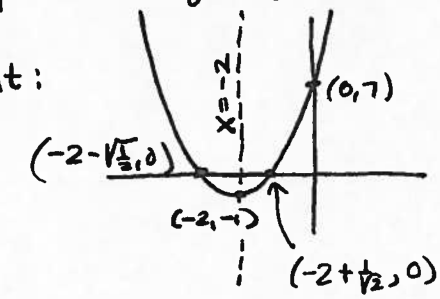
d. $0 = 2(x-2)^2 - 1 \Rightarrow 1 = 2(x-2)^2$; $\frac{1}{2} = (x-2)^2$; $\pm\sqrt{\frac{1}{2}} = x - 2$

$\Rightarrow x = 2 \pm \sqrt{\frac{1}{2}}$

e. $x=0 \Rightarrow y = 7.$ f. 

so range $= [-1, \infty)$.

g. 


g.alt: 


alt: a. $\frac{1}{2}g(x) = x^2+4x+4+7/2-4 \Rightarrow g(x) = 2(x+2)^2-1.$

b. vertex is $(-2,-1)$. c axis $x=-2$

d. $0 = g(x) \Rightarrow x = -2 \pm \sqrt{1/2}$ e. $x=0 \Rightarrow y=7.$ f. range $= [-1, \infty)$