A vector space is a nonempty set $V$ of objects, called vectors, on which are defined two operations, called addition and multiplication by scalars (real numbers), subject to the ten axioms (or rules) listed below. The axioms must hold for all vectors $u$, $v$, and $w$ in $V$ and for all scalars $c$ and $d$.

1. The sum of $u$ and $v$, denoted by $u + v$, is in $V$.
2. $u + v = v + u$.
3. $(u + v) + w = u + (v + w)$.
4. There is a zero vector $0$ in $V$ such that $u + 0 = u$.
5. For each $u$ in $V$, there is a vector $-u$ in $V$ such that $u + (-u) = 0$.
6. The scalar multiple of $u$ by $c$, denoted by $cu$, is in $V$.
7. $c(u + v) = cu + cv$.
8. $(c + d)u = cu + du$.
9. $c(du) = (cd)u$.
10. $1u = u$.

Using only these axioms, one can show that the zero vector in Axiom 4 is unique, and the vector $-u$, called the negative of $u$, in Axiom 5 is unique for each $u$ in $V$. See Exercises 25 and 26. Proofs of the following simple facts are also outlined in the exercises:

For each $u$ in $V$ and scalar $c$,

\[
0u = 0 \quad (1)
\]

\[
c0 = 0 \quad (2)
\]

\[-u = (-1)u \quad (3)
\]

**Example 1** The spaces $\mathbb{R}^n$, where $n \geq 1$, are the premier examples of vector spaces. The geometric intuition developed for $\mathbb{R}^3$ will help you understand and visualize many concepts throughout the chapter.

**Example 2** Let $V$ be the set of all arrows (directed line segments) in three-dimensional space, with two arrows regarded as equal if they have the same length and point in the same direction. Define addition by the parallelogram rule (from Section 1.3), and for each $v$ in $V$, define $cv$ to be the arrow whose length is $|c|$ times the length of $v$, pointing in the same direction as $v$ if $c \geq 0$ and otherwise pointing in the opposite direction. (See Fig. 1.) Show that $V$ is a vector space. This space is a common model in physical problems for various forces.

\[\text{Technically, } V \text{ is a real vector space. All of the theory in this chapter also holds for a complex vector space in which the scalars are complex numbers. We will look at this briefly in Chapter 5. Until then, all scalars are assumed to be real.}\]