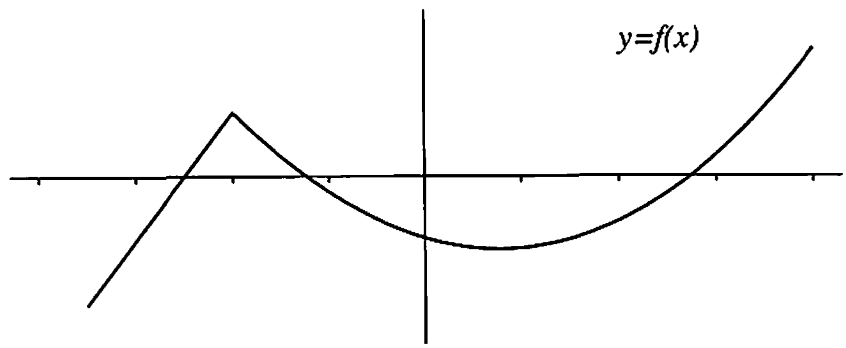
Math 120–001 (Kunkle), Exam 2. Name: ______________________
100 pts, 50 minutes Feb. 20, 2009

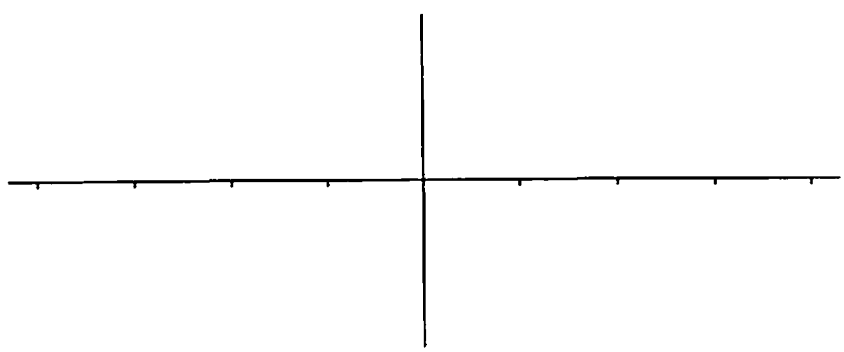
No calculators, notes, books, or any outside materials.
You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$. If you wish, you can leave unfinished arithmetic in your final answers.
Unless otherwise indicated, **supporting work will be required on every problem**; one-word answers, or answers which simply restate the question, will receive no credit.

---

1 (20 pts). Use the definition of derivative to find $f'(x)$ if $f(x) = \sqrt{3-5x}$
You can use the rules of differentiation from Chapter 3 to check your work, but, to receive credit on this problem, you must take the derivative using its definition.

2 (10 pts). The graph of $y = f(x)$ appears below. On the axes provided, sketch the graph of $y = f'(x)$.





3 (12 pts). Find the derivatives of the following functions. You are not required to simplify your answers.

a. $f(x) = \dfrac{4x}{5x^2} + 10\sqrt[5]{x^7}$

b. $g(x) = x^2 \cot x$

4 (32 pts). Find the derivatives of the following functions. You are not required to simplify your answers.

a. $\nu(x) = \dfrac{5+\cos x}{2e^x+5}$

b. $h(x) = \sec(e^x)$

c. $\eta(x) = \sqrt{\tan^{-1}(4x)}$

d. $k(x) = 4x^{\sqrt{2}}\tan x \cos^{-1} x$

5 (12 pts). Find the derivative of $\xi(x) = (1+3x)^{-11}\sin(\sin^{-1} x)$

You are not required to simplify your answer.

6 (14 pts). Find an equation of the line tangent to the curve $y^2 + x - e^{xy} = 8$ at the point $(0, 3)$.

① $f(x) = \sqrt{5-3x}$ $\quad f'(x) = \lim_{h\to 0} \dfrac{\sqrt{5-3(x+h)} - \sqrt{5-3x}}{h}$ $\Big\}$ mult. by conjugate

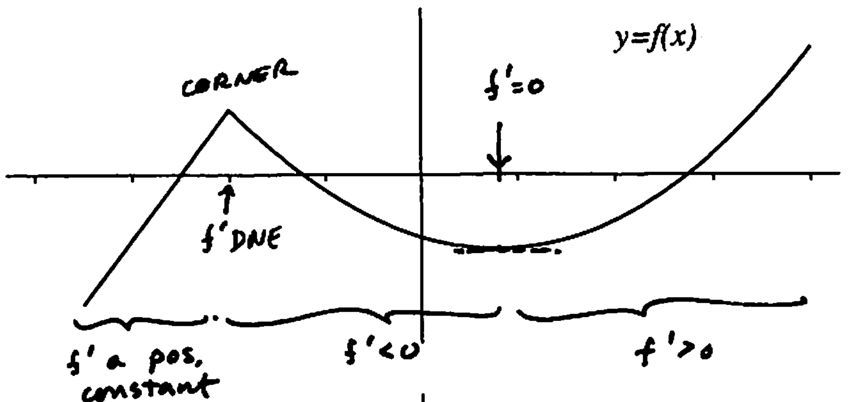
$$= \lim_{h\to 0} \frac{\left(\sqrt{5-3(x+h)} - \sqrt{5-3x}\right)\left(\sqrt{5-3(x+h)} + \sqrt{5-3x}\right)}{h\left(\sqrt{5-3(x+h)} + \sqrt{5-3x}\right)}$$

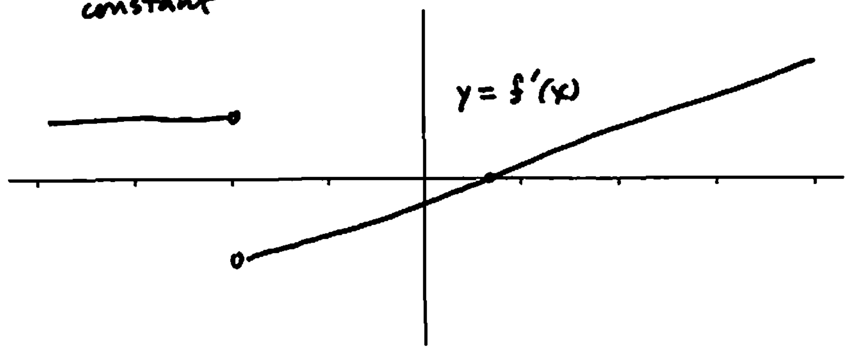
$$= \lim_{h\to 0} \frac{5-3(x+h) - (5-3x)}{h\left(\sqrt{5-3(x+h)} + \sqrt{5-3x}\right)}$$ ← use parentheses!

$$= \lim_{h\to 0} \frac{-3h}{h\left(\sqrt{\quad "\quad} + \sqrt{\quad "\quad}\right)} = \lim_{h\to 0} \frac{-3}{\left(\sqrt{5-3(x+h)} + \sqrt{5-3x}\right)}$$

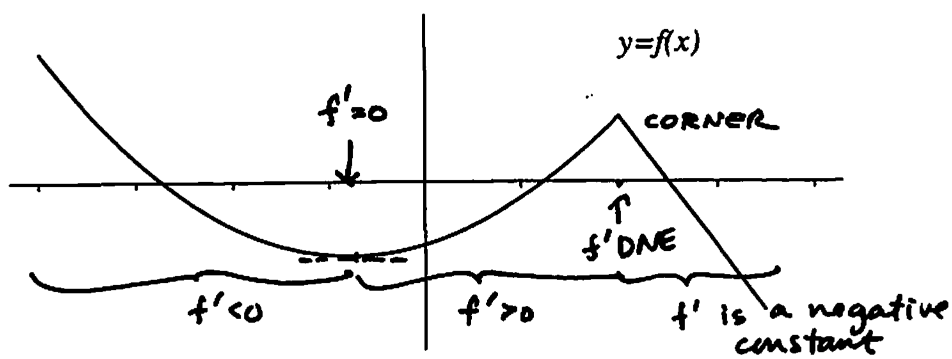
$$= \frac{-3}{2\sqrt{5-3x}}.$$ Alt. form. Interchange 5, 3. $f'(x) = \dfrac{-5}{2\sqrt{3-5x}}$

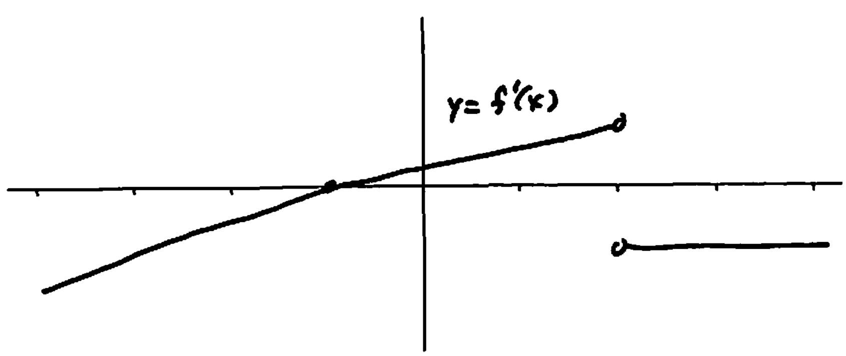
2 (10 pts). The graph of $y = f(x)$ appears below. On the axes provided, sketch the graph of $y = f'(x)$.





2 (10 pts). The graph of $y = f(x)$ appears below. On the axes provided, sketch the graph of $y = f'(x)$.





③ a. $f(x) = \dfrac{4x}{5x^2} + 10\sqrt[5]{x^7} = \dfrac{4}{5}x^{-1} + 10x^{7/5}$.

$f'(x) = -\dfrac{4}{5}x^{-2} + 10 \cdot \dfrac{7}{5}x^{2/5}$ done

Alt. form: $f(x) = \dfrac{5x}{4x^2} + 21\sqrt[7]{x^5} = \dfrac{5}{4}x^{-1} + 21x^{5/7}$.

$f'(x) = -\dfrac{5}{4}x^{-2} + 21 \cdot \dfrac{5}{7}x^{-2/7}$ done

b. $g(x) = x^2 \cot x$. Use product rule.

$g'(x) = (x^2)'\cot x + x^2(\cot x)' = 2x\cot x - x^2\csc^2 x$.

Alt form $g(x) = x^3 \tan x$. $g'(x) = 3x^2 \tan x + x^3 \sec^2 x$.

④ a. $v(x) = \dfrac{5+\cos x}{2e^x+5}$. Try quotient rule:

$$v'(x) = \frac{(5+\cos x)'(2e^x+5) - (5+\cos x)(2e^x-5)'}{(2e^x+5)^2}$$

$$= \frac{(-\sin x)(2e^x+5) - (5+\cos x)(2e^x)}{(2e^x+5)^2}.$$

Alt form:

$v(x) = \dfrac{2e^x+3}{3+\sin x}$. $v'(x) = \dfrac{2e^x(3+\sin x) - (2e^x+3)(\cos x)}{(3+\sin x)^2}$.

b. $h(x) = \csc(e^x)$. Chain Rule. Remember $\dfrac{d}{d\theta}\csc\theta = -\csc\theta\cot\theta$.

$h'(x) = -\csc(e^x)\cot(e^x)\cdot(e^x)' = -e^x \csc e^x \cot e^x$.

alt: $h(x) = \sec(e^x)$. $h'(x) = e^x \sec e^x \tan e^x$.

c. $\eta(x) = \left(\tan^{-1}(4x)\right)^{1/2}$ 2 applications of Chain Rule:

$$\eta'(x) = \frac{1}{2}\left(\tan^{-1}(4x)\right)^{-1/2} \frac{1}{1+(4x)^2} \cdot 4 = \frac{2}{(1+16x^2)\sqrt{\tan^{-1} 4x}}$$

(done)

alt. form: $\eta(x) = \left(\tan^{-1} 3x\right)^{1/2}$.

$$\eta'(x) = \frac{1}{2}\left(\tan^{-1} 3x\right)^{-1/2} \frac{1}{1+(3x)^2} \cdot 3$$ done

d. $k(x) = 4x^{\sqrt{2}} \cot x \sin^{-1} x$. product of 3 functions.

$$k'(x) = \left(4x^{\sqrt{2}}\right)' \cot x \sin^{-1} x + 4x^{\sqrt{2}}(\cot x)' \sin^{-1} x + 4x^{\sqrt{2}} \cot x \left(\sin^{-1} x\right)'$$

$$= 4\sqrt{2}\, x^{\sqrt{2}-1} \cot x \sin^{-1} x - 4x^{\sqrt{2}} \csc^2 x \sin^{-1} x + 4x^{\sqrt{2}} \cot x \frac{1}{\sqrt{1-x^2}}.$$

alt. $k = 4x^{\sqrt{2}} \tan x \cos^{-1} x$.

$$k'(x) = 4\sqrt{2}\, x^{\sqrt{2}-1} \tan x \cos^{-1} x + 4x^{\sqrt{2}} \sec^2 x \cos^{-1} x + 4x^{\sqrt{2}} \tan x \frac{-1}{\sqrt{1-x^2}}.$$

5 $\xi(x) = (1+3x)^{-11} \sin(\sin^{-1} x)$. 2ND factor simplifies to x, so

$\xi(x) = \underbrace{x\,(1+3x)^{-11}}_{\text{product}}$. $\xi'(x) = 1\,(1+3x)^{-11} - x \cdot 11\,(1+3x)^{-12} \cdot 3$.

alt. form $\xi(x) = (1+3x)^{-12} \overbrace{\cos(\cos^{-1} x)}^{=x} = x\,(1+3x)^{-12}$.

$\xi'(x) = (1+3x)^{-12} - x \cdot 12\,(1+3x)^{-13} \cdot 3$ done

You can differentiate $g(x)$ without simplifying first.
Here's what it looks like:

$g(x) = (1+3x)^{-11}\sin(\sin^{-1}x)$ ← product

$g'(x) = \left((1+3x)^{-11}\right)'\sin(\sin^{-1}x) + (1+3x)^{-11}\left(\sin(\sin^{-1}x)\right)'$

$= -11(1+3x)^{-12}\cdot 3\cdot\sin(\sin^{-1}x) + (1+3x)^{-11}\cos(\sin^{-1}x)\cdot\frac{1}{\sqrt{1-x^2}}$. done

alt form:

$g'(x) = -12(1+3x)^{-13}\cdot 3\cos(\cos^{-1}x) + (1+3x)^{-12}(-\sin(\cos^{-1}x))\,\frac{-1}{\sqrt{1-x^2}}$ done

⑥ Differentiate both sides of $y^2 + x - e^{xy} = 8$ w.r.t. x:

$2y\frac{dy}{dx} + 1 - e^{xy}\left(1\cdot y + x\cdot\frac{dy}{dx}\right) = 0.$

Need slope only @ $(0,3)$, so plug these in before solving.

$6\frac{dy}{dx} + 1 - 1(3+0) = 0$; $6\frac{dy}{dx} - 2 = 0$; $\frac{dy}{dx} = 1/3$

line is $y-3 = 1/3(x-0)$. done

alt. form, eqn is $y^2 - x - e^{xy} = 3$; pt. is $(0,2)$

$2y\frac{dy}{dx} - 1 - e^{xy}\left(y + x\frac{dy}{dx}\right) = 0$. Plug in $(0,2)$ & find $\frac{dy}{dx}$.

$4\frac{dy}{dx} - 1 - 1(2+0) = 0$; $4\frac{dy}{dx} - 3 = 0$; $\frac{dy}{dx} = 3/4$.

line is $y-2 = \frac{3}{4}(x-0)$. done